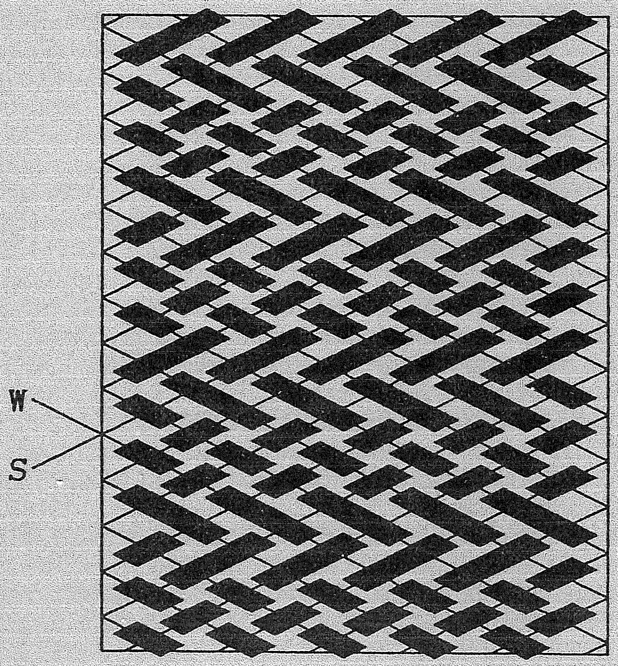
THE BRAIDING OF ROW-CODED REGULAR KNOTS



A.G. Schaake
J.C. Turner

9

Topics in Braiding Theory and Practice

Pamphlet No.9

OTHER PAMPHLETS IN THIS SERIES :

Pamphlet No. 1	**Introducing Grid-diagrams in Braiding**
Pamphlet No. 2	**Edge Lacing --- The Double Cordovan Stitch**
Pamphlet No. 3	**Braiding Application --- Horse Halter**
Pamphlet No. 4	**The Regular Knot Tree and Enlargement Processes**
Pamphlet No. 5	**An Introduction to Flat Braids**
Pamphlet No. 6	**An Introduction to Evolution Processes (Part 1)**
Pamphlet No. 7	**The Braiding of Column-Coded Regular Knots**
Pamphlet No. 8	**Knots — Facts and Fallacies**

These pamphlets may be obtained from:

Rams Skull Press
Mr. Ron Edwards
12 Fairyland Road
Kuranda, Qld 4872
Australia

Ph: (070) 937 474
FAX: 070 514 484

Footrope Knots
Des & Liz Pawson
501 Wherstead Road
Ipswich
Suffolk, IP2 8LL
England

The Braiding of Row-coded Regular Knots

A.G. Schaake
J.C. Turner

Abstract:

A general method for obtaining the information which enables a braider to braid any Row-coded Regular Knot is fully discussed. Refinements, such as an aid in carrying out a continual check on the braiding progress; and the construction of a Row-coded Regular Knot by starting at the midpoint of its string-length, are dealt with.

Copyright © -- A.G. Schaake, J.C. Turner.

All rights reserved.

Free use of the material in this pamphlet is encouraged, provided that the source is fully acknowledged.

January, 1993.

CONTENTS

page

INTRODUCTION 1

THE ALGORITHM-DIAGRAMS FOR ROW-CODED REGULAR KNOTS 2

PARTLY UPWARDS BRAIDING AND PARTLY DOWNWARDS BRAIDING ... 26

A BRAIDING START AT THE RIGHT-HAND BIGHT-BOUNDARY 43

REFERENCES 46

Introduction

In Pamphlet No.7 we gave a simple method for obtaining the half-cycle braiding-algorithms for Column-coded Regular Knots by means of an **algorithm-diagram.** In that pamphlet, we fully discussed, with worked examples, the outstanding advantages of this method over any other in providing for:

(1) Any of the two braiding-directions.
(2) Any of the two bight-boundary starts.
(3) Braiding the knot partly upwards and partly downwards (a very desirable procedure for big knots especially).
(4) Providing an indicator when and how each even-numbered half-cycle crosses the Standing End (an important feature for checking the braiding progress).

In order to obtain for Column-coded Regular Knots the most simple algorithm-diagrams which incorporate the properties under items (1) and (3) above, it is necessary **not** to indicate the crossing-types with "**o**" and "**u**", but instead by their respective codings "/" and "\"; an important point that should **not** be overlooked.

Since it is not possible to provide a Row-coded Regular Knot with a single algorithm-diagram that contains all the properties listed in the items (1) to (4) above, the coding-indicators "/" and "\" do not serve any specific merit for these knots. Hence for Row-coded Regular Knots it is better to use in their algorithm-diagrams the "**o**" and "**u**" crossing-type indicators instead.

Braiders who have studied Pamphlet No.7 thoroughly should not encounter any difficulties with the procedures as set out in this pamphlet since the basic 'ingredients', such as the **complementary cyclic bight number-scheme**, and the even and odd half-cycle numbers with their associated bight-numbers "i", are constructed and calculated in exactly the same manner as in Pamplet No.7. Hence, to avoid unnecessary repetition, we shall assume that the reader has mastered the procedures in that pamphlet.

In this pamphlet we shall concern ourselves mainly with the braiding of Row-coded Regular Knots in which the braiding starts at the left-hand bight-boundary. Any braider who prefers to start at the right-hand bight-boundary should not encounter any difficulties in the drawing up of the associated algorithm-diagrams. We shall only very briefly discuss the case in which an algorithm-diagram associated with a left-hand bight-boundary start is modified into an algorithm-diagram associated with a right-hand bight-boundary start.

As in Pamphlet No.7, we shall present several worked examples in order to clarify the various procedures involved. To use the algorithm-diagrams efficiently, it is advantageous to construct them out of two separate parts, which can then be used like a slide-rule; this is clearly shown in some of the discussed examples. In order to facilitate this, a set of six sheets which contain the 'two part' algorithm-diagrams of the examples is supplied with this pamphlet. They may be photo-copied and the reader can then cut out the central sliding part, along the black lines, for each algorithm-diagram and use it as shown in the examples. Furthermore, the last of these six sheets also contains a general skeleton algorithm-diagram which can be used as a template for the construction of a required algorithm-diagram. In this diagram the **o**'s and **u**'s are to be entered above and below their short vertical position indicators.

The Algorithm-diagrams for Row-coded Regular Knots

As mentioned in the Introduction, we shall assume, in order to avoid unnecessary repetition, that the reader is familiar with the contents of Pamphlet No. 7 (see Ref. [3]).

In this Chapter we shall discuss, with the aid of worked examples, the construction of algorithm-diagrams which are associated with:

(1) A braiding-start at the left-hand bight-boundary.

(2) Braiding in the upwards direction. Hence the 1st half-cycle runs from lower-left to upper-right, followed by the 2nd half-cycle running from lower-right to upper-left.

There are a total of $(p-1)$ crossings along each half-cycle in a finished Regular Knot with p-parts. Unlike Column-coded Regular Knots, the Row-coded Regular Knots do not have, for these $(p-1)$ crossings, the same consecutive coding-pattern for each odd-numbered half-cycle, and neither do they have the same consecutive coding-pattern for each even-numbered half-cycle. However, the consecutive coding-pattern along a half-cycle consists of a repeating coding-pattern which can readily be derived from the coding-pattern along the 1st half-cycle of the completed Row-coded Regular Knot. Since we have two sets of half-cycles, an odd-numbered set (each one running from lower-left to upper-right) and an even-numbered set (each one running from lower-right to upper-left), we require two coding-sequences (one for each set of half-cycles) in our algorithm-diagram. We shall explain the derivation of these two coding-sequences with the aid of Fig. 1, where for clarity the complementary cyclic bight-number sequence has been omitted in the algorithm-diagram.

To obtain these two coding-sequences, we set off, on a horizontal line, a number of equally spaced dots. We change the second dot from the left into a star, and do the same with the p^{th} dot from the left. The 1st dot from the left represents the left-hand bight-boundary of the Row-coded Regular Knot. The left-hand star represents the first crossing on the 1st half-cycle as well as the last crossing on the 2nd half-cycle. The right-hand star represents the last crossing on the 1st half-cycle as well as the first crossing on the 2nd half-cycle. The $(p+1)^{th}$ dot from the left represents the right-hand bight-boundary of the Row-coded Regular Knot.

Below these marked positions, starting at the **left-hand star**, we set off the consecutive coding-pattern along the 1st half-cycle from lower-left to upper-right. This provides all positions from the left-hand star to, and including, the right-hand star with a coding entry. Hence in Fig. 1 we obtain the coding-sequence:

$$u-u-o-o-u-o-u-o-o-u-u-o-u-o-u-u$$

From this coding-sequence determine the repeating coding-sequence. Hence in Fig. 1 we obtain the repeating coding-sequence:

$$u-u-o-o-u-o-u-o-o-u-u-o-u-o$$

Below this repeating coding-sequence, we set off vertical line segments at the even positions from the left. Hence the first vertical line segment is below the coding entry at the left-hand star. The number of vertical line segments in the repeating coding-sequence is called the **repeat** "r". Hence in Fig. 1 we obtain 7 vertical line segments, consequently the repeat $r=7$.

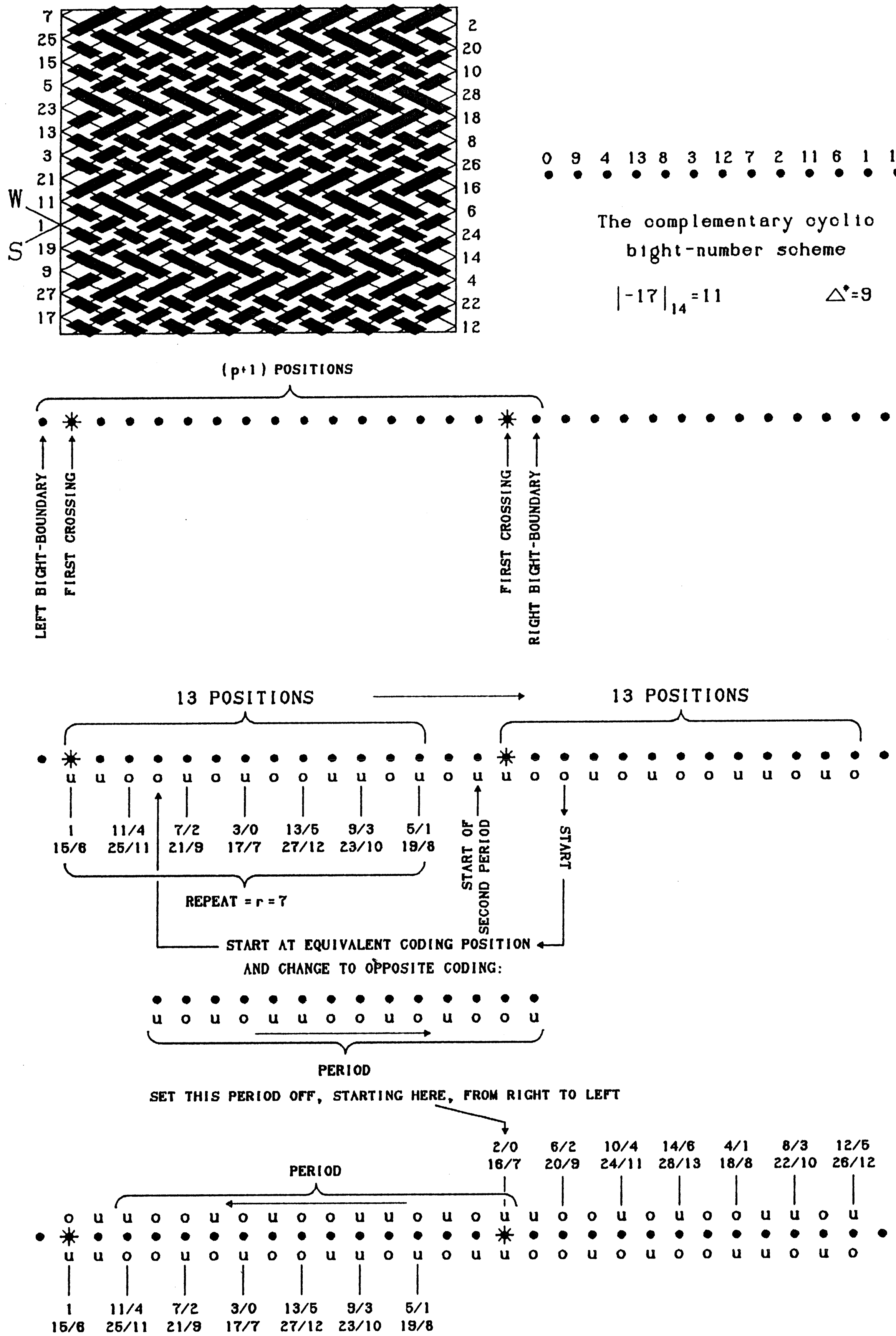

7
25
15
5
23
13
3
21
11
1
19
9
27
17
W
S
2
20
10
28
18
8
26
16
6
24
14
4
22
12
0 9 4 13 8 3 12 7 2 11 6 1 10 5
The complementary cyclic bight-number scheme
|-17|₁₄=11
Δ*=9
(p+1) POSITIONS
LEFT BIGHT-BOUNDARY
FIRST CROSSING
FIRST CROSSING
RIGHT BIGHT-BOUNDARY
13 POSITIONS
13 POSITIONS
u u o o u o u o o u u o u o u u o o u o u o o u u o u o
1 15/6
11/4 25/11
7/2 21/9
3/0 17/7
13/5 27/12
9/3 23/10
5/1 19/8
REPEAT = r = 7
START OF SECOND PERIOD
START
START AT EQUIVALENT CODING POSITION
AND CHANGE TO OPPOSITE CODING:
u o u o u u o o u o u o o u
PERIOD
SET THIS PERIOD OFF, STARTING HERE, FROM RIGHT TO LEFT
PERIOD
2/0 16/7
6/2 20/9
10/4 24/11
14/6 28/13
4/1 18/8
8/3 22/10
12/5 26/12
o u u o o u o u o o u u o u o u u o o u o u o o u u o u
u u o o u o u o o u u o u o u u o o u o u o o u u o u o
1 15/6
11/4 25/11
7/2 21/9
3/0 17/7
13/5 27/12
9/3 23/10
5/1 19/8

FIG. 1

We must ensure that there are $(2r-1)$ of the regularly spaced dots to the right of the right-hand star. Hence in Fig. 1 we require $2r-1 = 2 \times 7 - 1 = 13$ dots to the right of the right-hand star.

Continue coding these further positions to the right of the right-hand star, **except the last one**, in compliance with the repeating coding-pattern. Hence in Fig. 1 we continue to the right of the right-hand star with the coding-sequence:

$$o - o - u - o - u - o - o - u - u - o - u - o$$

Take the coding-sequence to the right, starting at the second position to the right of the right-hand star. Hence in Fig. 1 this coding-sequence is:

$$o - u - o - u - o - o - u - u - o - u - o$$

Find the equivalent coding-sequence from the left-hand side, and take from the beginning of this sequence the first $2r$ consecutive codings to the right. Change these codings to the opposite form and set them off below a set of $2r$ equally spaced dots. Hence in Fig. 1 we obtain the new repeating coding-pattern:

$$u - o - u - o - u - u - o - o - u - o - u - o - o - u$$

With this new repeating coding-pattern, **being read from left to right**, we construct the coding-sequence along the 2^{nd} half-cycle in the following manner: Starting at the right-hand star position, set off above the marked positions, **from right to left**, this new repeating coding sequence. Further, enter the appropriate coding, in accordance with the repeating coding-pattern, above the remaining positions, except the first and last positions from the left.

We have now obtained the two coding-sequences of the algorithm-diagram: the one below the marked positions is associated with the odd-numbered half-cycles, and the one above the marked positions is associated with the even-numbered half-cycles.

The reader will note that the odd-numbered half-cycles, read from left to right, are associated with the entries below the marked positions, and that the even-numbered half-cycles, read from right to left, are associated with the entries above the marked positions. The reason for doing this will become obvious later when we consider the algorithm-diagrams for braiding the Row-coded Regular Knots partly upwards and partly downwards.

Next we draw, above the coding entries, r vertical line segments, starting at the right-hand star position and continuing to the right at consecutive second positions.

As mentioned earlier, the Row-coded Regular Knots do not have the same consecutive coding-pattern for each of the odd-numbered half-cycles, nor for each of the even-numbered half-cycles. In order to use in the algorithm-diagram one coding-sequence for the odd-numbered half-cycles, and one coding-sequence for the even-numbered half-cycles, we must position the first crossing on each half-cycle in such a way along its respective algorithm-diagram coding-sequence that the actual half-cycle coding-sequence is obtained. This can readily be achieved by starting each half-cycle at the appropriate vertical line segment; hence we have to assign to each vertical line segment its associated half-cycle numbers. There are two methods of doing this, which we shall indicate by **method A*** and **method B***. For both methods, we assign to each of the two sets of r vertical line segments sequential numbers, ranging from 1 to r. Hence if we indicate such a vertical line segment number by j, then $1 \leq j \leq r$. The vertical line segments associated with the two star positions always carry the number $j = 1$.

The consecutive numbering of the vertical line segments starts with $j = 1$ and moves cyclically in the direction of the associated half-cycles.

Method A*:

Calculate the value $|p|_r$. This value is used in the assignation of half-cycle numbers to the vertical line segments. The procedure involved, which we shall illustrate with the aid of the example in Figs. 1, 2, is similar to the one used in the construction of the complementary cyclic bight-number scheme.

In Figs. 1, 2 we obtain for $|p|_r$:

$$|17|_7 = 3.$$

The odd-numbered half-cycles are associated with the lower left-hand set of $r = 7$ vertical line segments.

The vertical line segment $j = 1$ is always associated with half-cycle 1, which is always a free run.

Count 3 (the value of $|p|_r$) vertical line segments in consecutive (cyclic) order, and assign to the last vertical line segment the half-cycle 3 (with $i = 0$), indicated by 3/0.

Count a further 3 vertical line segments in consecutive (cyclic) order, and assign to the last vertical line segment the half-cycle 5 (with $i = 1$), indicated by 5/1.

Count a further 3 vertical line segments in consecutive (cyclic) order, and assign to the last vertical line segment the half-cycle 7 (with $i = 2$), indicated by 7/2.

Keep repeating this process until all the $r = 7$ vertical line segments have been assigned one half-cycle number. These half-cycle numbers are called **the basic half-cycle numbers**, and their associated bight-numbers are called **the basic bight-numbers.** We shall have obtained in consecutive (cyclic) order the entries:

1 , 11/4 , 7/2 , 3/0 , 13/5 , 9/3 , 5/1

Note that each entry, except the first (the 1st half-cycle), has, besides a half-cycle number, a bight-number. However, for convenience we assign mentally the hypothetical bight-number −1 to the first one.

We assign to each of the vertical line segments further half-cycle number/bight-number sets by adding $2r$ (=14) to the half-cycle number and by adding r (=7) to the bight-number respectively. Repeat this process with each newly obtained half-cycle number/bight-number set. Discard any half-cycle number/bight-number set in which the half-cycle number is greater than $2b$ $(= 2 \times 14 = 28)$.

The reader will now see the reason for assigning mentally the hypothetical bight-number value of −1 to the 1st half-cycle.

The even-numbered half-cycles are associated with the upper right-hand set of $r = 7$ vertical line segments.

The vertical line segment $j = 1$ is always associated with half-cycle 2 and bight-number 0.

Count 3 (the value of $|p|_r$) vertical line segments in consecutive (cyclic) order, and assign to the last vertical line segment the half-cycle 4 (with $i = 1$), indicated by 4/1.

Count a further 3 vertical line segments in consecutive (cyclic) order, and assign to the last vertical line segment the half-cycle 6 (with $i = 2$), indicated by 6/2.

Count a further 3 vertical line segments in consecutive (cyclic) order, and assign to the last vertical line segment the half-cycle 8 (with $i = 3$), indicated by 8/3.

Keep repeating this process until all the $r = 7$ vertical line segments have been assigned one half-cycle number. These half-cycle numbers are again called **the basic half-cycle numbers**, and their associated bight-numbers are again called **the basic bight-numbers.** We shall have obtained in consecutive (cyclic) order the entries:

2/0 , 12/5 , 8/3 , 4/1 , 14/6 , 10/4 , 6/2

As before, each entry has, besides a half-cycle number, a bight-number.

In exactly the same way as before, we assign to each of the vertical line segments further half-cycle number/bight-number sets by adding $2r$ (=14) to the half-cycle number and by adding r (=7) to the bight-number respectively. Repeat this process with each newly obtained half-cycle number/bight-number set. Discard any half-cycle number/bight-number set in which the half-cycle number is greater than $2b$ $(= 2 \times 14 = 28)$.

The complementary cyclic bight-number scheme is set off along the "dot"-positions in exactly the same way as for the Column-coded Regular Knots (see Ref. [3]). Note however, that the Left to Right half-cycles (the odd-numbered ones) are now associated with the entries below the "dot"-positions, and that the Right to Left half-cycles (the even-numbered ones) are now associated with the entries above the "dot"-positions.

Method B*:

Instead of the counting procedure of method A*, we can calculate the basic half-cycle number/bight-number sets associated with the various vertical line segments. These calculations can readily be performed with the value for Δh. This value is calculated in the following way:

$$|\Delta i \cdot p|_b = 1$$
$$\Delta h = 2 \cdot |\Delta i|_r$$

If we indicate the basic half-cycle number belonging to the vertical line segment j by $h_{0,j}$, and the basic bight-number by $i_{0,j}$ then:

$$\begin{aligned}
h_{0,j+1} &= |h_{0,j} + \Delta h|_{2r} \qquad \text{with } 1 \le j \le r-1\,; \\
i_{0,j+1} &= |i_{0,j} + \Delta i|_r \qquad \text{with } 1 \le j \le r-1\,; \\
h_{0,1} &= 1 \qquad \text{for odd half-cycles}\,; \\
i_{0,1} &= -1 \text{ (hypothetically)} \qquad \text{for odd half-cycles}\,; \\
h_{0,1} &= 2 \qquad \text{for even half-cycles}\,; \\
i_{0,1} &= 0 \qquad \text{for even half-cycles}\,;
\end{aligned}$$

Thus in Figs. 1, 2 we get: $|\Delta i \cdot p|_b = |\Delta i \times 17|_{14} = |\Delta i \times 3|_{14} = 1$. The smallest positive integer for Δi which gives a solution is 5. Hence with $\Delta i = 5$ we obtain $\Delta h = 2 \cdot |\Delta i|_r = 2 \times 5 = 10$.

With these values for Δh and Δi we can now calculate the basic half-cycle number/basic bight-number sets for the odd and even half-cycles. Thus we obtain in consecutive cyclic order the sets as found under Method A* above.

The assignation of further half-cycle number/bight-number sets to each of the vertical line segments is accomplished in the manner as described under method A* above. This is expressed in formula form as follows:

$$h_{n,j} = h_{0,j} + 2nr \quad \text{with } n = 1, 2, 3, \cdots \quad \text{and} \quad h_{n,j} \le 2b\,.$$

$$i_{n,j} = i_{0,j} + nr \quad \text{with } n = 1, 2, 3, \cdots \quad \text{and} \quad \begin{cases} i_{n,j} \le b-2 \text{ for } h_{n,j} \text{ is odd;} \\ i_{n,j} \le b-1 \text{ for } h_{n,j} \text{ is even.} \end{cases}$$

To use the algorithm-diagram in braiding the knot concerned, we have to make the strip with dots and bight-numbers sliding in order to line the star-positions up with the various vertical line segments. Furthermore, for convenience sake, we delete the $j = x$ entries.

Method B*:

Instead of the counting procedure of method A*, we can calculate the basic half-cycle number/bight-number sets associated with the various vertical line segments. These calculations can readily be performed with the value for Δh. This value is calculated in the following way:

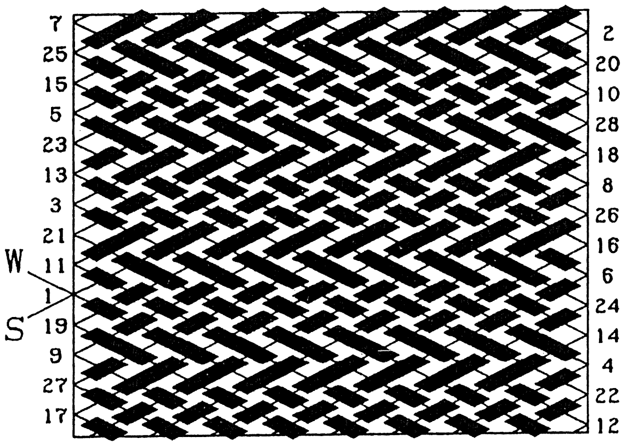
$$|\Delta i \cdot p|_b = 1 \qquad \text{hence } \left||\Delta i|_r \cdot p\right|_r = 1\,;$$
$$\Delta h = 2 \cdot |\Delta i|_r\,.$$

If we indicate the basic half-cycle number belonging to the vertical line segment j by $h_{0,j}$, and the basic bight-number by $i_{0,j}$ then:

$$h_{0,j+1} = |h_{0,j} + \Delta h|_{2r} \qquad \text{with } 1 \le j \le r-1\,;$$
$$i_{0,j+1} = |i_{0,j} + \Delta i|_r \qquad \text{with } 1 \le j \le r-1\,;$$
$$h_{0,1} = 1 \qquad \text{for odd half-cycles}\,;$$
$$i_{0,1} = -1 \text{ (hypothetically)} \qquad \text{for odd half-cycles}\,;$$
$$h_{0,1} = 2 \qquad \text{for even half-cycles}\,;$$
$$i_{0,1} = 0 \qquad \text{for even half-cycles}\,;$$

Thus in Figs. 1,2 we get: $|\Delta i \cdot p|_b = |\Delta i \times 17|_{14} = |\Delta i \times 3|_{14} = 1$. The smallest positive integer for Δi which gives a solution is 5. Hence with $\Delta i = 5$ we obtain $\Delta h = 2 \cdot |\Delta i|_r = 2 \cdot |5|_7 = 2 \times 5 = 10$.

With these values for Δh and $|\Delta i|_r$ (or Δi) we can now calculate the basic half-cycle number/basic bight-number sets for the odd and even half-cycles. Thus we obtain in consecutive cyclic order the sets as found under Method A* above.



0 9 4 13 8 3 12 7 2 11 6 1 10 5

● ● ● ● ● ● ● ● ● ● ● ● ● ●

The complementary cyclic bight-number scheme

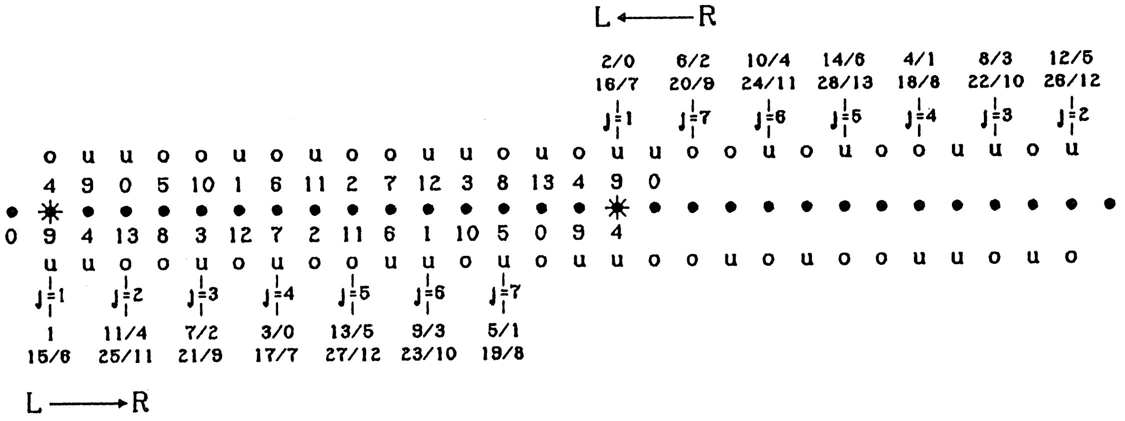
$|-17|_{14}=11$ $\Delta^{*}=9$

$|p|_r = |17|_7 = 3$ $\Delta h = 10$ $h_{0,j+1} = \left|h_{0,j} + \Delta h\right|_{2r}$ with $1 \leq j \leq r-1$

$h_{0,1}=1$ for odd half-cycles

$h_{0,1}=2$ for even half-cycles

$h_{n,j} = h_{0,j} + 2nr$ $\begin{cases}\text{with } n=1,2,3,\dots\\ \text{and } h_{n,j} \leq 2b\end{cases}$



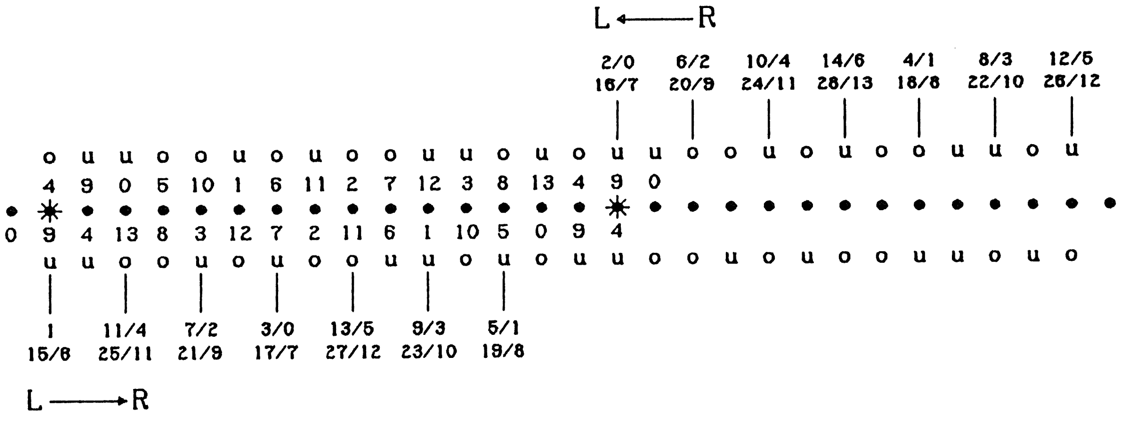


FIG.2

The reader is reminded (see Introduction) that a set of six sheets, containing the algorithm-diagrams of the knots discussed, is supplied with this pamphlet. The central sliding bight-number strips of the algorithm-diagrams can be cut out along the horizontal black lines and used as intended.

The information required in the braiding of any chosen half-cycle is obtained from the algorithm-diagram (for odd-numbered half-cycles after positioning the left-hand star under the appropriate vertical line segment associated with the odd half-cycle concerned, and for even-numbered half-cycles after positioning the right-hand star under the appropriate vertical line segment associated with the even half-cycle concerned) in exactly the same way as for Column-coded Regular Knots (see Ref. [3]). Successive steps in this process are depicted in Figs. $2, 2A, 2B, 2C$:

Half-cycle 1: This half-cycle is always a free run. Thus we obtain:

1. L $\longrightarrow$ R : free run .

Half-cycle 2: In the algorithm-diagram at the bottom of Fig. 2 the right-hand star is lined up with half-cycle 2 (bight-number 0). Starting at the right-hand star and ending at the left-hand star, read from right to left the successive codings associated with the bight-number(s) 0. Since bight-number 0 belongs to this half-cycle, these crossings are made by this half-cycle with the Standing End. Thus in our usual notation (see Ref. [3]) we obtain:

2. $(i = 0)$ R $\longrightarrow$ L : $(s)u$.

Half-cycle 3: In the first algorithm-diagram of Fig. $2A$ the left-hand star is lined up with half-cycle 3 (bight-number 0). Starting at the left-hand star and ending at the right-hand star, read from left to right the successive codings associated with the bight-number(s) 0. Thus we obtain:

3. $(i = 0)$ L $\longrightarrow$ R : o .

Half-cycle 4: In the second algorithm-diagram of Fig. $2A$ the right-hand star is lined up with half-cycle 4 (bight-number 1). Starting at the right-hand star and ending at the left-hand star, read from right to left the successive codings associated with the bight-numbers 0 and 1. Since bight-number 1 belongs to this half-cycle, the crossing(s) associated with bight-number(s) 1 are made by this half-cycle with the Standing End. Thus in our usual notation (see Ref. [3]) we obtain:

4. $(i = 1)$ R $\longrightarrow$ L : $(s,1)2u$.

Half-cycle 5: In the third algorithm-diagram of Fig. $2A$ the left-hand star is lined up with half-cycle 5 (bight-number 1). Starting at the left-hand star and ending at the right-hand star, read from left to right the successive codings associated with the bight-numbers 0 and 1. Thus we obtain:

5. $(i = 1)$ L $\longrightarrow$ R : $2o$.

Half-cycle 6: In the fourth algorithm-diagram of Fig. $2A$ the right-hand star is lined up with half-cycle 6 (bight-number 2). Starting at the right-hand star and ending at the left-hand star, read from right to left the successive codings associated with the bight-numbers 0, 1 and 2. Since bight-number 2 belongs to this half-cycle, the crossing(s) associated with bight-number(s) 2 are made by this half-cycle with the Standing End. Thus in our usual notation (see Ref. [3]) we obtain:

6. $(i = 2)$ R $\longrightarrow$ L : $(s,1)2u - o$.

Half-cycle 7: In the first algorithm-diagram of Fig. $2B$ the left-hand star is lined up with half-cycle 7 (bight-number 2). Starting at the left-hand star and ending at the right-hand star, read from left to right the successive codings associated with the bight-numbers $0, 1$ and 2. Thus we obtain:

7. $(i=2)$ L $\longrightarrow$ R : $o - u - o$.

Half-cycle 8: In the second algorithm-diagram of Fig. $2B$ the right-hand star is lined up with half-cycle 8 (bight-number 3). Starting at the right-hand star and ending at the left-hand star, read from right to left the successive codings associated with the bight-numbers $0, 1, 2$ and 3. Since bight-number 3 belongs to this half-cycle, the crossing(s) associated with bight-number(s) 3 are made by this half-cycle with the Standing End. Thus in our usual notation (see Ref. [3]) we obtain:

8. $(i=3)$ R $\longrightarrow$ L : $(s)u - o - u - o$.

Half-cycle 9: In the third algorithm-diagram of Fig. $2B$ the left-hand star is lined up with half-cycle 9 (bight-number 3). Starting at the left-hand star and ending at the right-hand star, read from left to right the successive codings associated with the bight-numbers $0, 1, 2$ and 3. Thus we obtain:

9. $(i=3)$ L $\longrightarrow$ R : $u - o + 2u$.

Half-cycle 10: In the fourth algorithm-diagram of Fig. $2B$ the right-hand star is lined up with half-cycle 10 (bight-number 4). Starting at the right-hand star and ending at the left-hand star, read from right to left the successive codings associated with the bight-numbers $0, 1, 2, 3$ and 4. Since bight-number 4 belongs to this half-cycle, the crossing(s) associated with bight-number(s) 4 are made by this half-cycle with the Standing End. Thus in our usual notation (see Ref. [3]) we obtain:

10. $(i=4)$ R $\longrightarrow$ L : $(s)o - u - (3,s)4o$.

Half-cycle 11: In the first algorithm-diagram of Fig. $2C$ the left-hand star is lined up with half-cycle 11 (bight-number 4). Starting at the left-hand star and ending at the right-hand star, read from left to right the successive codings associated with the bight-numbers $0, 1, 2, 3$ and 4. Thus we obtain:

11. $(i=4)$ L $\longrightarrow$ R : $o - 4u + o$.

Half-cycle 12: In the second algorithm-diagram of Fig. $2C$ the right-hand star is lined up with half-cycle 12 (bight-number 5). Starting at the right-hand star and ending at the left-hand star, read from right to left the successive codings associated with the bight-numbers $0, 1, 2, 3, 4$ and 5. Since bight-number 5 belongs to this half-cycle, the crossing(s) associated with bight-number(s) 5 are made by this half-cycle with the Standing End. Thus in our usual notation (see Ref. [3]) we obtain:

12. $(i=5)$ R $\longrightarrow$ L : $4o - (s)u - 2o$.

Half-cycle 13: In the third algorithm-diagram of Fig. $2C$ the left-hand star is lined up with half-cycle 13 (bight-number 5). Starting at the left-hand star and ending at the right-hand star, read from left to right the successive codings associated with the bight-numbers $0, 1, 2, 3, 4$ and 5. Thus we obtain:

13. $(i=5)$ L $\longrightarrow$ R : $5u - o - u$.

Half-cycle 14: In the fourth algorithm-diagram of Fig. $2C$ the right-hand star is lined up with half-cycle 14 (bight-number 6). Starting at the right-hand star and ending at the left-hand star, read from right to left the successive codings associated with the bight-numbers $0, 1, 2, 3, 4, 5$ and 6. Since bight-number 6 belongs to this half-cycle, the crossing(s) associated with bight-number(s) 6 are made by this half-cycle with the Standing End. Thus in our usual notation (see Ref. [3]) we obtain:

14. $(i=6)$ R $\longrightarrow$ L : $(3,s)4o - u - 3o$.

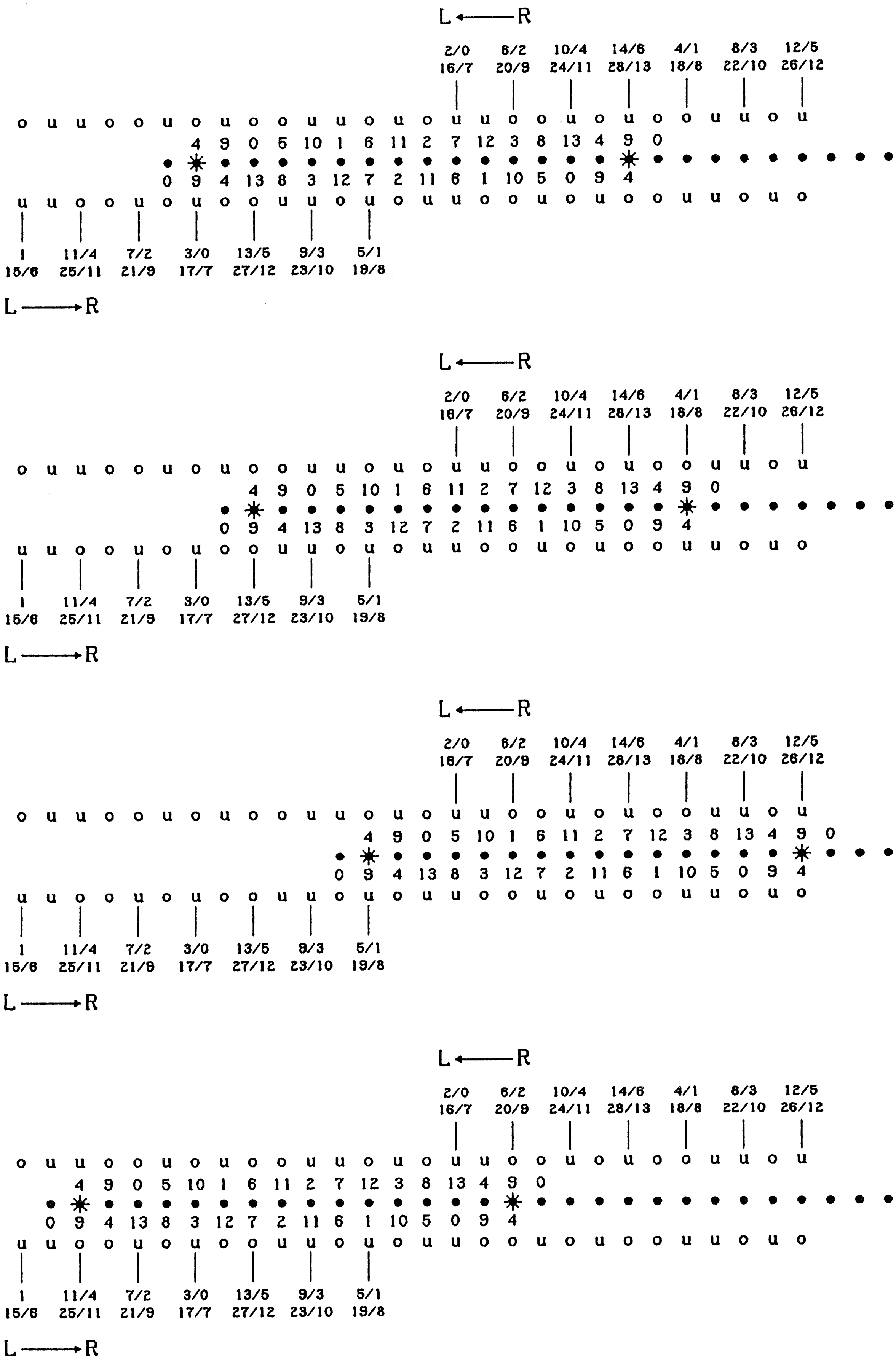

L ← R
2/0 16/7
6/2 20/9
10/4 24/11
14/6 28/13
4/1 18/8
8/3 22/10
12/5 26/12
o u u o o u o u o o u u o u o u u o o u o u o o u u o u
4 9 0 5 10 1 6 11 2 7 12 3 8 13 4 9 0
0 9 4 13 8 3 12 7 2 11 6 1 10 5 0 9 4
u u o o u o u o o u u o u o u u o o u o u o o u u o u o
1 15/6
11/4 25/11
7/2 21/9
3/0 17/7
13/5 27/12
9/3 23/10
5/1 19/8
L → R
L ← R
2/0 16/7
6/2 20/9
10/4 24/11
14/6 28/13
4/1 18/8
8/3 22/10
12/5 26/12
o u u o o u o u o o u u o u o u u o o u o u o o u u o u
4 9 0 5 10 1 6 11 2 7 12 3 8 13 4 9 0
0 9 4 13 8 3 12 7 2 11 6 1 10 5 0 9 4
u u o o u o u o o u u o u o u u o o u o u o o u u o u o
1 15/6
11/4 25/11
7/2 21/9
3/0 17/7
13/5 27/12
9/3 23/10
5/1 19/8
L → R
L ← R
2/0 16/7
6/2 20/9
10/4 24/11
14/6 28/13
4/1 18/8
8/3 22/10
12/5 26/12
o u u o o u o u o o u u o u o u u o o u o u o o u u o u
4 9 0 5 10 1 6 11 2 7 12 3 8 13 4 9 0
0 9 4 13 8 3 12 7 2 11 6 1 10 5 0 9 4
u u o o u o u o o u u o u o u u o o u o u o o u u o u o
1 15/6
11/4 25/11
7/2 21/9
3/0 17/7
13/5 27/12
9/3 23/10
5/1 19/8
L → R
L ← R
2/0 16/7
6/2 20/9
10/4 24/11
14/6 28/13
4/1 18/8
8/3 22/10
12/5 26/12
o u u o o u o u o o u u o u o u u o o u o u o o u u o u
4 9 0 5 10 1 6 11 2 7 12 3 8 13 4 9 0
0 9 4 13 8 3 12 7 2 11 6 1 10 5 0 9 4
u u o o u o u o o u u o u o u u o o u o u o o u u o u o
1 15/6
11/4 25/11
7/2 21/9
3/0 17/7
13/5 27/12
9/3 23/10
5/1 19/8
L → R

FIG. 2A

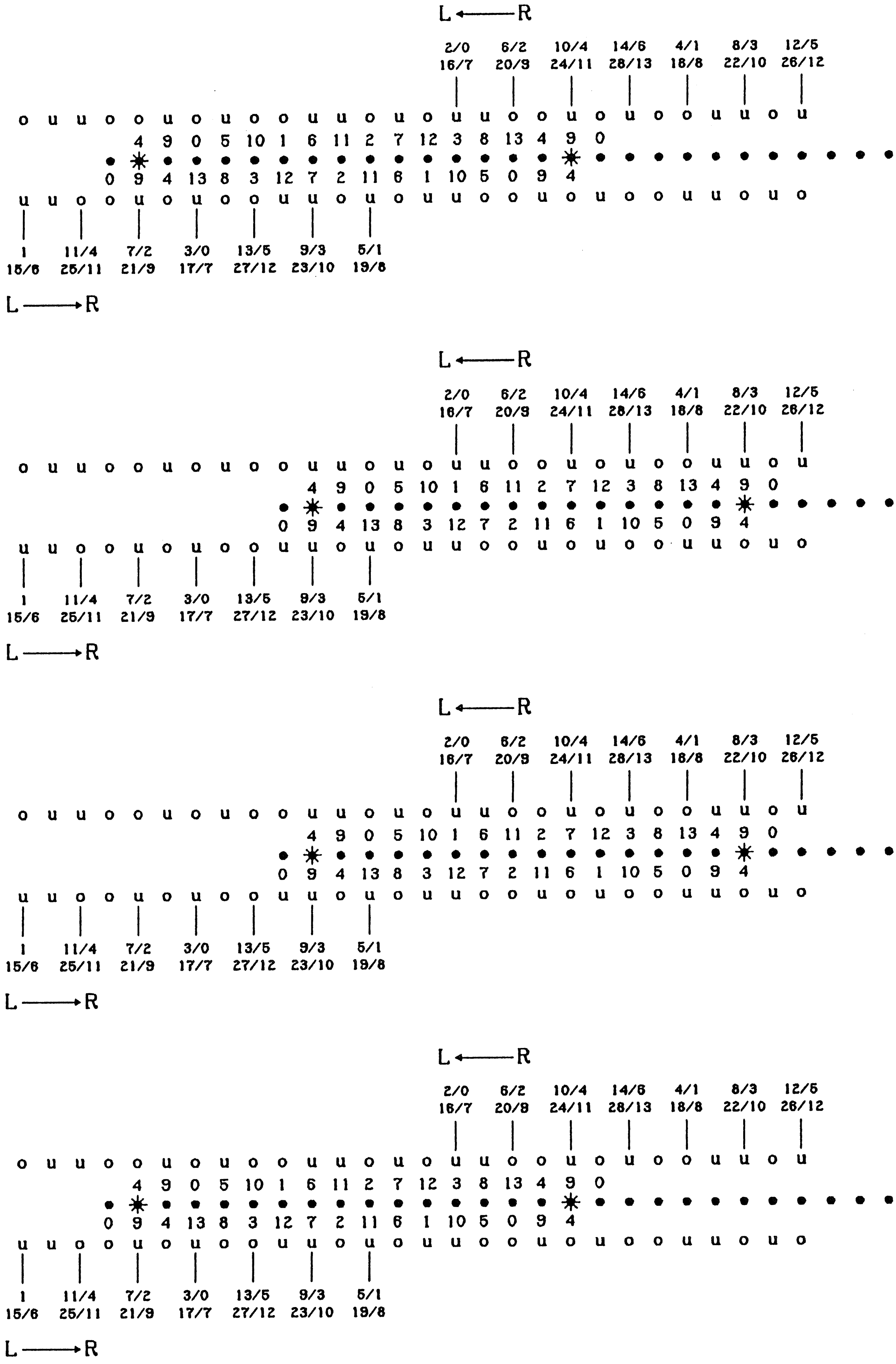

L←R
2/0 16/7
6/2 20/9
10/4 24/11
14/6 28/13
4/1 18/8
8/3 22/10
12/5 26/12
o u u o o u o u o o u u o u o u u o o u o u o o u u o u
4 9 0 5 10 1 6 11 2 7 12 3 8 13 4 9 0
0 9 4 13 8 3 12 7 2 11 6 1 10 5 0 9 4
u u o o u o u o o u u o u o u u o o u o u o o u u o u o
1 15/6
11/4 25/11
7/2 21/9
3/0 17/7
13/5 27/12
9/3 23/10
5/1 19/8
L→R
L←R
2/0 16/7
6/2 20/9
10/4 24/11
14/6 28/13
4/1 18/8
8/3 22/10
12/5 26/12
o u u o o u o u o o u u o u o u u o o u o u o o u u o u
4 9 0 5 10 1 6 11 2 7 12 3 8 13 4 9 0
0 9 4 13 8 3 12 7 2 11 6 1 10 5 0 9 4
u u o o u o u o o u u o u o u u o o u o u o o u u o u o
1 15/6
11/4 25/11
7/2 21/9
3/0 17/7
13/5 27/12
9/3 23/10
5/1 19/8
L→R
L←R
2/0 16/7
6/2 20/9
10/4 24/11
14/6 28/13
4/1 18/8
8/3 22/10
12/5 26/12
o u u o o u o u o o u u o u o u u o o u o u o o u u o u
4 9 0 5 10 1 6 11 2 7 12 3 8 13 4 9 0
0 9 4 13 8 3 12 7 2 11 6 1 10 5 0 9 4
u u o o u o u o o u u o u o u u o o u o u o o u u o u o
1 15/6
11/4 25/11
7/2 21/9
3/0 17/7
13/5 27/12
9/3 23/10
5/1 19/8
L→R
L←R
2/0 16/7
6/2 20/9
10/4 24/11
14/6 28/13
4/1 18/8
8/3 22/10
12/5 26/12
o u u o o u o u o o u u o u o u u o o u o u o o u u o u
4 9 0 5 10 1 6 11 2 7 12 3 8 13 4 9 0
0 9 4 13 8 3 12 7 2 11 6 1 10 5 0 9 4
u u o o u o u o o u u o u o u u o o u o u o o u u o u o
1 15/6
11/4 25/11
7/2 21/9
3/0 17/7
13/5 27/12
9/3 23/10
5/1 19/8
L→R

FIG.2B

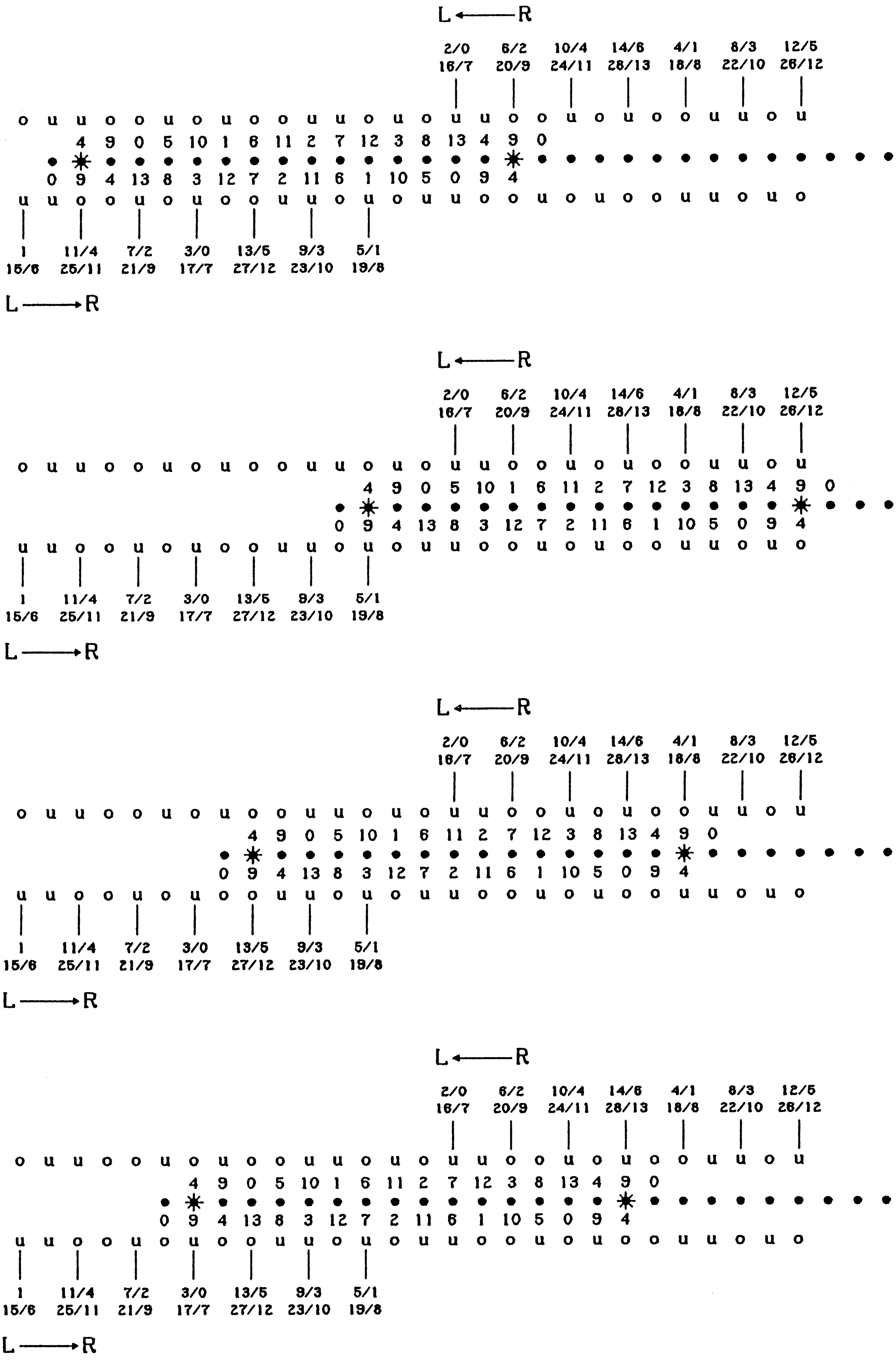

L←R
2/0 16/7
6/2 20/9
10/4 24/11
14/6 28/13
4/1 18/8
8/3 22/10
12/5 26/12
o u u o o u o u o o u u o u o u u o o u o u o o u u o u
4 9 0 5 10 1 6 11 2 7 12 3 8 13 4 9 0
0 9 4 13 8 3 12 7 2 11 6 1 10 5 0 9 4
u u o o u o u o o u u o u o u u o o u o u o o u u o u o
1 15/6
11/4 25/11
7/2 21/9
3/0 17/7
13/5 27/12
9/3 23/10
5/1 19/8
L→R
L←R
2/0 16/7
6/2 20/9
10/4 24/11
14/6 28/13
4/1 18/8
8/3 22/10
12/5 26/12
o u u o o u o u o o u u o u o u u o o u o u o o u u o u
4 9 0 5 10 1 6 11 2 7 12 3 8 13 4 9 0
0 9 4 13 8 3 12 7 2 11 6 1 10 5 0 9 4
u u o o u o u o o u u o u o u u o o u o u o o u u o u o
1 15/6
11/4 25/11
7/2 21/9
3/0 17/7
13/5 27/12
9/3 23/10
5/1 19/8
L→R
L←R
2/0 16/7
6/2 20/9
10/4 24/11
14/6 28/13
4/1 18/8
8/3 22/10
12/5 26/12
o u u o o u o u o o u u o u o u u o o u o u o o u u o u
4 9 0 5 10 1 6 11 2 7 12 3 8 13 4 9 0
0 9 4 13 8 3 12 7 2 11 6 1 10 5 0 9 4
u u o o u o u o o u u o u o u u o o u o u o o u u o u o
1 15/6
11/4 25/11
7/2 21/9
3/0 17/7
13/5 27/12
9/3 23/10
5/1 19/8
L→R
L←R
2/0 16/7
6/2 20/9
10/4 24/11
14/6 28/13
4/1 18/8
8/3 22/10
12/5 26/12
o u u o o u o u o o u u o u o u u o o u o u o o u u o u
4 9 0 5 10 1 6 11 2 7 12 3 8 13 4 9 0
0 9 4 13 8 3 12 7 2 11 6 1 10 5 0 9 4
u u o o u o u o o u u o u o u u o o u o u o o u u o u o
1 15/6
11/4 25/11
7/2 21/9
3/0 17/7
13/5 27/12
9/3 23/10
5/1 19/8
L→R

FIG.2C

The reader will have observed that we only need to slide the ce ral sliding bight-number strip of the algorithm-diagram into r (therefore in our examp into 7) different positions to enable us to read off the half-cycle algorithms of a R w-coded Regular Knot.

The complete half-cycle algorithm-table for the above knot is a follows (see also Ref. [1], pages 82 and 104, No. 39):

1. $\mathrm{L} \longrightarrow \mathrm{R}$: free run.
2. $(i=0)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $(s)u$.
3. $(i=0)$ $\mathrm{L} \longrightarrow \mathrm{R}$: o .
4. $(i=1)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $(s,1)2u$.
5. $(i=1)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $2o$.
6. $(i=2)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $(s,1)2u-o$.
7. $(i=2)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $o-u-o$.
8. $(i=3)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $(s)u-o-u-o$.
9. $(i=3)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $u-o-2u$.
10. $(i=4)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $(s)o-u-(3,s)4o$.
11. $(i=4)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $o-4u-o$.
12. $(i=5)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $4o-(s)u-2o$.
13. $(i=5)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $5u-o-u$.
14. $(i=6)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $(3,s)4o-u-3o$.
15. $(i=6)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $2u-o-3u-o-u$.
16. $(i=7)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $o-u-(s,2)3o-u-o-u-o$.
17. $(i=7)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $o-2u-o-u-o-u-2o$.
18. $(i=8)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $(1,s)2o-u-o-u-o-3u-o$.
19. $(i=8)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $o-u-o-u-3o-u-2o$.
20. $(i=9)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $(s)o-u-o-3u-o-2u-(1,s)2o-$
21. $(i=9)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $u-3o-u-2o-u-2o-u-o$.
22. $(i=10)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $2u-o-2u-o-2u-(s)o-u-o-2$.
23. $(i=10)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $u-2o-u-3o-u-2o-2u-o$.
24. $(i=11)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $u-o-3u-o-(s,1)2u-2o-u-o-$ $-o$.
25. $(i=11)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $3o-u-o-2u-o-u-o-2u-2o$.
26. $(i=12)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $u-o-u-(1,s)2o-u-o-u-2o-$ $-o-u-o$.
27. $(i=12)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $o-u-o-u-o-2u-2o-u-o-u$ $2o-u$.
28. $(i=13)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $u-o-(s)u-2o-2u-o-u-o-2u$ $2o-u-o$.

The next worked example is associated with the Row-coded R ular Knot whose grid-diagram is depicted in Fig. 3. After constructing its algorithm-d gram (at bottom of Fig. 3), we can use it to derive the half-cycle algorithms required the braiding of this knot.

Half-cycle 1: This half-cycle is always a free run. Thus we obtain:

1. $\mathrm{L} \longrightarrow \mathrm{R}$: free run.

Half-cycle 2: In the algorithm-diagram at the bottom of Fig. 3 the ight-hand star is lined up with half-cycle 2 (bight-number 0). Starting at the right-ha l star and ending at the left-hand star, read from right to left the successive codings sociated with the bight-number(s) 0. Since bight-number 0 belongs to this half-cycle, ese crossings are made by this half-cycle with the Standing End. Thus in our usual no tion (see Ref. [3]) we obtain:

2. $(i=0)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $(s)o$.

Half-cycle 3: In the first algorithm-diagram of Fig. 3A the left-hand star is lined up with half-cycle 3 (bight-number 0). Starting at the left-hand star and ending at the right-hand star, read from left to right the successive codings associated with the bight-number(s) 0. Thus we obtain:

3. $(i=0)$ L $\longrightarrow$ R : o .

Half-cycle 4: In the second algorithm-diagram of Fig. 3A the right-hand star is lined up with half-cycle 4 (bight-number 1). Starting at the right-hand star and ending at the left-hand star, read from right to left the successive codings associated with the bight-numbers 0 and 1. Since bight-number 1 belongs to this half-cycle, the crossing(s) associated with bight-number(s) 1 are made by this half-cycle with the Standing End. Thus in our usual notation (see Ref. [3]) we obtain:

4. $(i=1)$ R $\longrightarrow$ L : $(s)u-o$.

Half-cycle 5: In the second algorithm-diagram of Fig. 3A the left-hand star is lined up with half-cycle 5 (bight-number 1). Starting at the left-hand star and ending at the right-hand star, read from left to right the successive codings associated with the bight-numbers 0 and 1. Thus we obtain:

5. $(i=1)$ L $\longrightarrow$ R : $o-u$.

Half-cycle 6: In the third (or first) algorithm-diagram of Fig. 3A the right-hand star is lined up with half-cycle 6 (bight-number 2). Starting at the right-hand star and ending at the left-hand star, read from right to left the successive codings associated with the bight-numbers 0, 1 and 2. Since bight-number 2 belongs to this half-cycle, the crossing(s) associated with bight-number(s) 2 are made by this half-cycle with the Standing End. Thus in our usual notation (see Ref. [3]) we obtain:

6. $(i=2)$ R $\longrightarrow$ L : $(s,1)2o-(1,s)2u$.

As mentioned previously, we only need to slide the central sliding bight-number strip of the algorithm-diagram into r (therefore in this example into 3) different positions to enable us to read off the half-cycle algorithms of a Row-coded Regular Knot.

The complete half-cycle algorithm-table for the above knot is as follows (see also Ref. [1], pages 79 and 100, No. 32):

1. L $\longrightarrow$ R : free run .
2. $(i=0)$ R $\longrightarrow$ L : $(s)o$.
3. $(i=0)$ L $\longrightarrow$ R : o .
4. $(i=1)$ R $\longrightarrow$ L : $(s)u-o$.
5. $(i=1)$ L $\longrightarrow$ R : $o-u$.
6. $(i=2)$ R $\longrightarrow$ L : $(s,1)2o-(1,s)2u$.
7. $(i=2)$ L $\longrightarrow$ R : $u-2o-u$.
8. $(i=3)$ R $\longrightarrow$ L : $u-(1,s,1)3o-u$.
9. $(i=3)$ L $\longrightarrow$ R : $o-u-2o-u$.
10. $(i=4)$ R $\longrightarrow$ L : $o-(s,2)3u-2o-(s)u$.
11. $(i=4)$ L $\longrightarrow$ R : $o-u-o-2u-2o$.
12. $(i=5)$ R $\longrightarrow$ L : $3o-u-(s)o-2u-o$.
13. $(i=5)$ L $\longrightarrow$ R : $u-2o-2u-o-u-o$.
14. $(i=6)$ R $\longrightarrow$ L : $2u-(s,2)3o-u-o-u-(1,s)2o$.
15. $(i=6)$ L $\longrightarrow$ R : $o-u-o-u-3o-2u-o$.
16. $(i=7)$ R $\longrightarrow$ L : $o-u-o-2u-o-(s)u-2o-2u$.
17. $(i=7)$ L $\longrightarrow$ R : $o-2u-o-u-o-2u-2o-u$.
18. $(i=8)$ R $\longrightarrow$ L : $2o-(1,s)2u-o-u-2o-2u-o-u$.

The reader will have observed that we only need to slide the central sliding bight-number strip of the algorithm-diagram into r (therefore in our example into 7) different positions to enable us to read off the half-cycle algorithms of a Row-coded Regular Knot.

The complete half-cycle algorithm-table for the above knot is as follows (see also Ref. [1], pages 82 and 104, No. 39):

1. L ⟶ R : free run .
2. $(i=0)$ R ⟶ L : $(s)u$.
3. $(i=0)$ L ⟶ R : o .
4. $(i=1)$ R ⟶ L : $(s,1)2u$.
5. $(i=1)$ L ⟶ R : $2o$.
6. $(i=2)$ R ⟶ L : $(s,1)2u-o$.
7. $(i=2)$ L ⟶ R : $o-u-o$.
8. $(i=3)$ R ⟶ L : $(s)u-o-u-o$.
9. $(i=3)$ L ⟶ R : $u-o-2u$.
10. $(i=4)$ R ⟶ L : $(s)o-u-(3,s)4o$.
11. $(i=4)$ L ⟶ R : $o-4u-o$.
12. $(i=5)$ R ⟶ L : $4o-(s)u-2o$.
13. $(i=5)$ L ⟶ R : $5u-o-u$.
14. $(i=6)$ R ⟶ L : $(3,s)4o-u-3o$.
15. $(i=6)$ L ⟶ R : $2u-o-3u-o-u$.
16. $(i=7)$ R ⟶ L : $o-u-(s,2)3o-u-o-u-o$.
17. $(i=7)$ L ⟶ R : $o-2u-o-u-o-u-2o$.
18. $(i=8)$ R ⟶ L : $(1,s)2o-u-o-u-o-3u-o$.
19. $(i=8)$ L ⟶ R : $o-u-o-u-3o-u-2o$.
20. $(i=9)$ R ⟶ L : $(s)o-u-o-3u-o-2u-(1,s)2o-u$.
21. $(i=9)$ L ⟶ R : $u-3o-u-2o-u-2o-u-o$.
22. $(i=10)$ R ⟶ L : $2u-o-2u-o-2u-(s)o-u-o-2u$.
23. $(i=10)$ L ⟶ R : $u-2o-u-3o-u-2o-2u-o$.
24. $(i=11)$ R ⟶ L : $u-o-3u-o-(s,1)2u-2o-u-o-u-o$.
25. $(i=11)$ L ⟶ R : $3o-u-o-2u-o-u-o-2u-2o$.
26. $(i=12)$ R ⟶ L : $u-o-u-(1,s)2o-u-o-u-2o-2u-o-u-o$.
27. $(i=12)$ L ⟶ R : $o-u-o-u-o-2u-2o-u-o-u-2o-u$.
28. $(i=13)$ R ⟶ L : $u-o-(s)u-2o-2u-o-u-o-2u-2o-u-o$.

The next worked example is associated with the Row-coded Regular Knot whose grid-diagram is depicted in Fig. 3. After constructing its algorithm-diagram (at bottom of Fig. 3), we can use it to derive the half-cycle algorithms required in the braiding of this knot.

Half-cycle 1: This half-cycle is always a free run. Thus we obtain:

1. L ⟶ R : free run .

Half-cycle 2: In the algorithm-diagram at the bottom of Fig. 3 the right-hand star is lined up with half-cycle 2 (bight-number 0). Starting at the right-hand star and ending at the left-hand star, read from right to left the successive codings associated with the bight-number(s) 0. Since bight-number 0 belongs to this half-cycle, these crossings are made by this half-cycle with the Standing End. Thus in our usual notation (see Ref. [3]) we obtain:

2. $(i=0)$ R ⟶ L : $(s)o$.

Half-cycle 3: In the first algorithm-diagram of Fig. 3A the left-hand star is lined up with half-cycle 3 (bight-number 0). Starting a the left-hand star and ending at the right-hand star, read from left to right the succe sive codings associated with the bight-number(s) 0. Thus we obtain:

3. $(i=0)$ $\mathrm{L} \longrightarrow \mathrm{R}:$ o .

Half-cycle 4: In the second algorithm-diagram of Fig. 3A the right-hand star is lined up with half-cycle 4 (bight-number 1). Starting at the right-hand star and ending at the left-hand star, read from right to left the successive codings associated with the bight-numbers 0 and 1. Since bight-number 1 belongs to this half-cycle, the crossing(s) associated with bight-number(s) 1 are made by this half-cycle with the Standing End. Thus in our usual notation (see Ref. [3]) we obtain:

4. $(i=1)$ $\mathrm{R} \longrightarrow \mathrm{L}:$ $(s)u-o$.

Half-cycle 5: In the second algorithm-diagram of Fig. 3A the left-hand star is lined up with half-cycle 5 (bight-number 1). Starting at the left-hand star and ending at the right-hand star, read from left to right the successive codings associated with the bight-numbers 0 and 1. Thus we obtain:

5. $(i=1)$ $\mathrm{L} \longrightarrow \mathrm{R}:$ $o-u$.

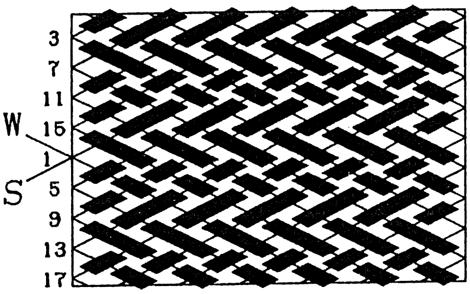
Half-cycle 6: In the third (or first) algorithm-diagram of Fig. 3A the right-hand star is lined up with half-cycle 6 (bight-number 2) Starting at the right-hand star and ending at the left-hand star, read from right to left the successive codings associated with the bight-numbers 0,1 and 2. Since bight-number 2 belongs to this half-cycle, the crossing(s) associated with bight-number(s) 2 are made by this half-cycle with the Standing End. Thus in our usual notation (see Ref. [3]) we obtain:

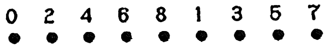
6. $(i=2)$ $\mathrm{R} \longrightarrow \mathrm{L}:$ $(s,1)2o-(1,s)2u$.

As mentioned previously, we only need to slide the central sliding bight-number strip of the algorithm-diagram into r (therefore in thi example into 3) different positions to enable us to read off the half-cycle algorithms o a Row-coded Regular Knot.

The complete half-cycle algorithm-table for the above knot is as follows (see also Ref. [1], pages 79 and 100, No. 32):

1. $\mathrm{L} \longrightarrow \mathrm{R}:$ free run .
2. $(i=0)$ $\mathrm{R} \longrightarrow \mathrm{L}:$ $(s)o$.
3. $(i=0)$ $\mathrm{L} \longrightarrow \mathrm{R}:$ o .
4. $(i=1)$ $\mathrm{R} \longrightarrow \mathrm{L}:$ $(s)u-o$.
5. $(i=1)$ $\mathrm{L} \longrightarrow \mathrm{R}:$ $o-u$.
6. $(i=2)$ $\mathrm{R} \longrightarrow \mathrm{L}:$ $(s,1)2o-(1,s)2u$.
7. $(i=2)$ $\mathrm{L} \longrightarrow \mathrm{R}:$ $u-2o-u$.
8. $(i=3)$ $\mathrm{R} \longrightarrow \mathrm{L}:$ $u-(1,s,1)3o-u$.
9. $(i=3)$ $\mathrm{L} \longrightarrow \mathrm{R}:$ $o-u-2o-u$.
10. $(i=4)$ $\mathrm{R} \longrightarrow \mathrm{L}:$ $o-(s,2)3u-2o-\ s)u$.
11. $(i=4)$ $\mathrm{L} \longrightarrow \mathrm{R}:$ $o-u-o-2u-2o$.
12. $(i=5)$ $\mathrm{R} \longrightarrow \mathrm{L}:$ $3o-u-(s)o-2u-o$.
13. $(i=5)$ $\mathrm{L} \longrightarrow \mathrm{R}:$ $u-2o-2u-o-u-o$.
14. $(i=6)$ $\mathrm{R} \longrightarrow \mathrm{L}:$ $2u-(s,2)3o-u-o-u-(1,s)2o$.
15. $(i=6)$ $\mathrm{L} \longrightarrow \mathrm{R}:$ $o-u-o-u-3o-2u-o$.
16. $(i=7)$ $\mathrm{R} \longrightarrow \mathrm{L}:$ $o-u-o-2u-o-(s)u-2o-2u$.
17. $(i=7)$ $\mathrm{L} \longrightarrow \mathrm{R}:$ $o-2u-o-u-o-2u-2o-u$.
18. $(i=8)$ $\mathrm{R} \longrightarrow \mathrm{L}:$ $2o-(1,s)2u-o-u-2o-2u-o-u$.





The complementary cyclic bight-number scheme

$|-13|_9=5$ $\triangle^*=2$

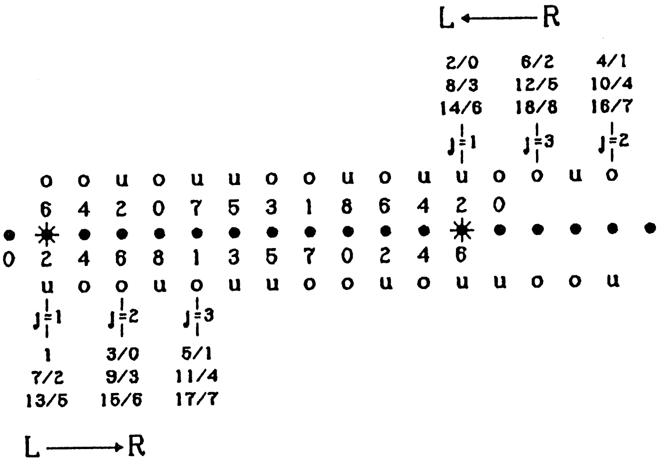
$|p|_r=|13|_3=1$ $\triangle h=2$

$h_{0,j+1}=\left|h_{0,j}+\triangle h\right|_{2r}$ with $1\leqq j\leqq r-1$

$h_{0,1}=1$ for odd half-cycles

$h_{0,1}=2$ for even half-cycles

$h_{n,j}=h_{0,j}+2nr \begin{cases}\text{with } n=1,2,3,\ldots \\ \text{and } h_{n,j}\leqq 2b\end{cases}$



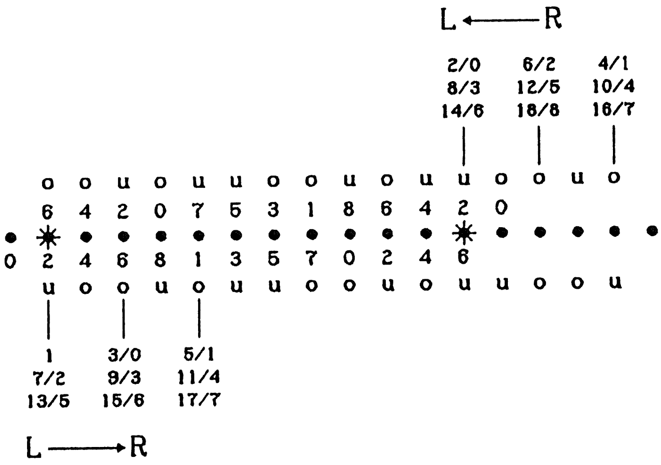


FIG.3

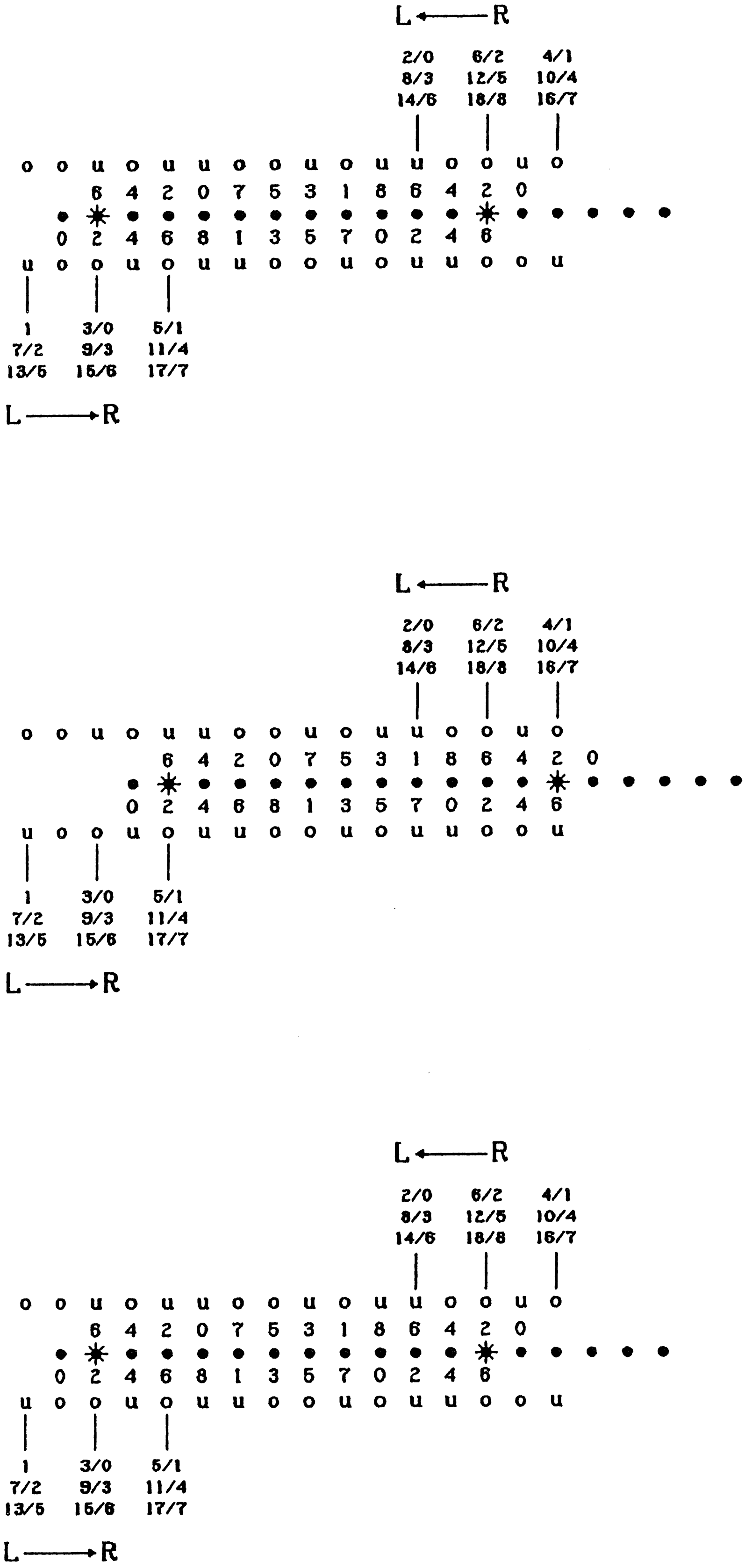

L←R
2/0
8/3
14/6
6/2
12/5
18/8
4/1
10/4
16/7
o o u o u u o o u o u u o o u o
6 4 2 0 7 5 3 1 8 6 4 2 0
0 2 4 6 8 1 3 5 7 0 2 4 6
u o o u o u u o o u o u u o o u
1
7/2
13/5
3/0
9/3
15/6
5/1
11/4
17/7
L→R
L←R
2/0
8/3
14/6
6/2
12/5
18/8
4/1
10/4
16/7
o o u o u u o o u o u u o o u o
6 4 2 0 7 5 3 1 8 6 4 2 0
0 2 4 6 8 1 3 5 7 0 2 4 6
u o o u o u u o o u o u u o o u
1
7/2
13/5
3/0
9/3
15/6
5/1
11/4
17/7
L→R
L←R
2/0
8/3
14/6
6/2
12/5
18/8
4/1
10/4
16/7
o o u o u u o o u o u u o o u o
6 4 2 0 7 5 3 1 8 6 4 2 0
0 2 4 6 8 1 3 5 7 0 2 4 6
u o o u o u u o o u o u u o o u
1
7/2
13/5
3/0
9/3
15/6
5/1
11/4
17/7
L→R

FIG.3A

A few more worked examples are given in Figs. 4, 5, 6 and 7. Below follow their respective half-cycle algorithm-tables.

Half-cycle algorithm-table for Fig. 4 (see also Ref. [1], pages 48–53):

1. $\mathrm{L} \longrightarrow \mathrm{R}$: free run .
2. $(i=0)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $(s)o-(s)u$.
3. $(i=0)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $o-u$.
4. $(i=1)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $(s,1)2u-(s,1)2o-(s)u$.
5. $(i=1)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $o-2u-2o$.
6. $(i=2)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $(1,s,1)3o-(1,s,1)3u-(1,s)2o$.
7. $(i=2)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $u-3o-3u-o$.
8. $(i=3)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $2u-(s)o-u-2o-(s)u-o-2u-(s)o$.
9. $(i=3)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $o-4u-4o-2u$.
10. $(i=4)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $2o-(1,s)2u-o-2u-(1,s)2o-u-2o-(1,s)2u$.
11. $(i=4)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $u-2o-u-2o-2u-o-2u-2o-u$.
12. $(i=5)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $2u-2o-(s,1)2u-2o-2u-(s,1)2o-2u-2o$.

Half-cycle algorithm-table for Fig. 5 (see also Ref. [1], pages 79 and 100, No. 31):

1. $\mathrm{L} \longrightarrow \mathrm{R}$: free run .
2. $(i=0)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $(s)o$.
3. $(i=0)$ $\mathrm{L} \longrightarrow \mathrm{R}$: u .
4. $(i=1)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $(s,1)2u$.
5. $(i=1)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $u-o$.
6. $(i=2)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $(s)u-o-u$.
7. $(i=2)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $o-2u$.
8. $(i=3)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $(s)o-2u-o$.
9. $(i=3)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $2u-o-u$.
10. $(i=4)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $(s,1)2u-o-2u-(s)o$.
11. $(i=4)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $u-o-2u-2o$.
12. $(i=5)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $o-2u-(1,s)2o-2u$.
13. $(i=5)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $2u-o-3u-o$.
14. $(i=6)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $u-o-(1,s,1)3u-3o$.
15. $(i=6)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $o-2u-3o-2u$.
16. $(i=7)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $2u-(s,2)3o-3u-o$.
17. $(i=7)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $u-2o-3u-3o$.
18. $(i=8)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $(1,s)2o-3u-3o-2u$.

Half-cycle algorithm-table for Fig. 6 :

1. $\mathrm{L} \longrightarrow \mathrm{R}$: free run .
2. $(i=0)$ $\mathrm{R} \longrightarrow \mathrm{L}$: free run .
3. $(i=0)$ $\mathrm{L} \longrightarrow \mathrm{R}$: free run .
4. $(i=1)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $(s)u$.
5. $(i=1)$ $\mathrm{L} \longrightarrow \mathrm{R}$: u .
6. $(i=2)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $o-(s)u$.
7. $(i=2)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $u-o$.
8. $(i=3)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $u-(1,s)2o$.
9. $(i=3)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $2o-u$.
10. $(i=4)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $u-o-u-(s)o$.
11. $(i=4)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $2u-2o$.
12. $(i=5)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $u-o-u-o-(s)u$.

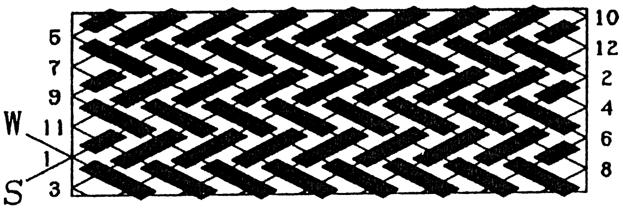
13. $(i = 5)$ L $\longrightarrow$ R : $u - o - 2u - o$.
14. $(i = 6)$ R $\longrightarrow$ L : $o - 2u - o - u - (s)o$.
15. $(i = 6)$ L $\longrightarrow$ R : $u - o - u - o - 2u$.
16. $(i = 7)$ R $\longrightarrow$ L : $u - 2o - 2u - o - (s)u$.
17. $(i = 7)$ L $\longrightarrow$ R : $2o - u - o - u - o - u$.
18. $(i = 8)$ R $\longrightarrow$ L : $u - o - u - 2o - 2u - (s)o$.
19. $(i = 8)$ L $\longrightarrow$ R : $2u - 2o - u - o - u - o$.
20. $(i = 9)$ R $\longrightarrow$ L : $u - o - u - o - u - 2o - (1, s)2u$.
21. $(i = 9)$ L $\longrightarrow$ R : $u - o - 2u - 2o - u - o - u$.
22. $(i = 10)$ R $\longrightarrow$ L : $o - 2u - o - u - o - u - 2o - (s)u$.
23. $(i = 10)$ L $\longrightarrow$ R : $u - o - u - o - 2u - 2o - u - o$.
24. $(i = 11)$ R $\longrightarrow$ L : $u - 2o - 2u - o - u - o - u - o$.

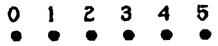
Half-cycle algorithm-table for Fig. 7 (see also Ref. [1], pages 81 and 104, No. 38):

1. L $\longrightarrow$ R : free run .
2. $(i = 0)$ R $\longrightarrow$ L : $(s)u$.
3. $(i = 0)$ L $\longrightarrow$ R : o .
4. $(i = 1)$ R $\longrightarrow$ L : $(s, 1, s)3u$.
5. $(i = 1)$ L $\longrightarrow$ R : $o - u - o$.
6. $(i = 2)$ R $\longrightarrow$ L : $o - (s)u - 2o - (s)u$.
7. $(i = 2)$ L $\longrightarrow$ R : $2u - o - 2u$.
8. $(i = 3)$ R $\longrightarrow$ L : $u - (1, s)2o - 2u - o$.
9. $(i = 3)$ L $\longrightarrow$ R : $2o - u - 3o$.
10. $(i = 4)$ R $\longrightarrow$ L : $(s)o - 3u - (1, s)2o - 2u$.
11. $(i = 4)$ L $\longrightarrow$ R : $o - u - 2o - u - o - u - o$.
12. $(i = 5)$ R $\longrightarrow$ L : $u - o - (s)u - o - 3u - o - (s)u - o$.
13. $(i = 5)$ L $\longrightarrow$ R : $3o - 2u - 4o - u$.
14. $(i = 6)$ R $\longrightarrow$ L : $o - (3, s)4u - o - u - o - 3u$.
15. $(i = 6)$ L $\longrightarrow$ R : $u - o - u - 3o - 2u - o - u - o$.
16. $(i = 7)$ R $\longrightarrow$ L : $(s, 1)2u - 2o - u - o - u - o - (s, 1)2u - 2o - u$.
17. $(i = 7)$ L $\longrightarrow$ R : $u - o - u - o - 2u - 2o - u - o - u - o - u$.
18. $(i = 8)$ R $\longrightarrow$ L : $o - 2u - (s)o - u - o - u - o - u - o - 2u - (s)o - u - o$.
19. $(i = 8)$ L $\longrightarrow$ R : $o - u - o - u - 3o - u - 2o - u - o - u - 2o$.
20. $(i = 9)$ R $\longrightarrow$ L : $u - o - 2u - o - u - (s)o - 2u - o - u - o - 2u - o - u$.

Although the algorithm-diagram for the Row-coded Regular knot of Fig. 7 can be constructed as depicted in Fig. 7*B*, such a construction form is in general not recommended. The braider has to be very careful not to make mistakes with such a construction form, and since this form cannot universally be applied, it is advisable not to use it. We have shown the form in this pamplet only in order to draw the reader's attention to it. Hence we have included Figs. 7*A* and 7*C* which respectively illustrate the remaining four positions of the central sliding bight-number strip for each construction form.

Instead of making the central bight-number strip sliding, we can continue with the respective bight-number patterns to the right. By assigning the appropriate replacement bight-numbers to each of the vertical line segments, we can then read off the respective half-cycle braiding algorithms by using the associated replacement bight-numbers within the applicable reading-boundaries. Since this method is more cumbersome and consequently cannot be recommended, we shall not discuss it further in this pamphlet.





The complementary cyclic bight-number scheme

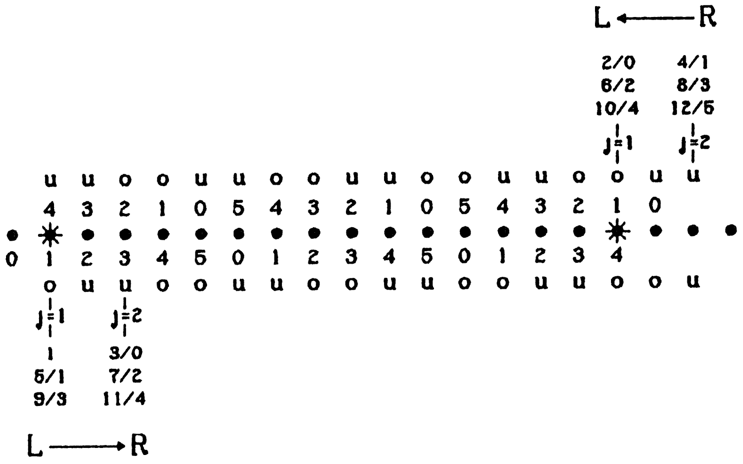
$|-17|_6 = 1$ $\Delta^* = 1$

$|p|_r = |17|_2 = 1$ $\Delta h = 2$ $h_{0,j+1} = \left|h_{0,j} + \Delta h\right|_{2r}$ with $1 \leq j \leq r-1$

$h_{0,1} = 1$ for odd half-cycles

$h_{0,1} = 2$ for even half-cycles

$h_{n,j} = h_{0,j} + 2nr$ $\begin{cases} \text{with } n = 1, 2, 3, \ldots \\ \text{and } h_{n,j} \leq 2b \end{cases}$



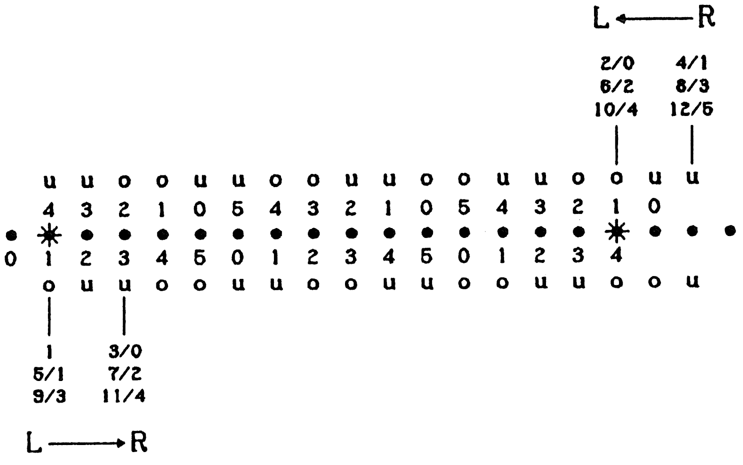
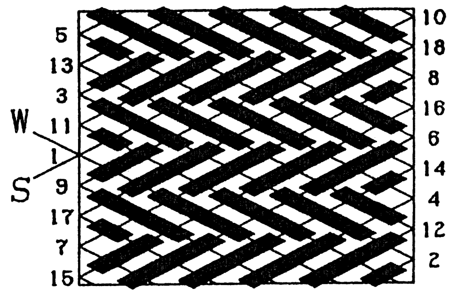
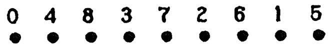


FIG.4





The complementary cyclic bight-number scheme

$|-11|_9=7$ $\triangle'=4$

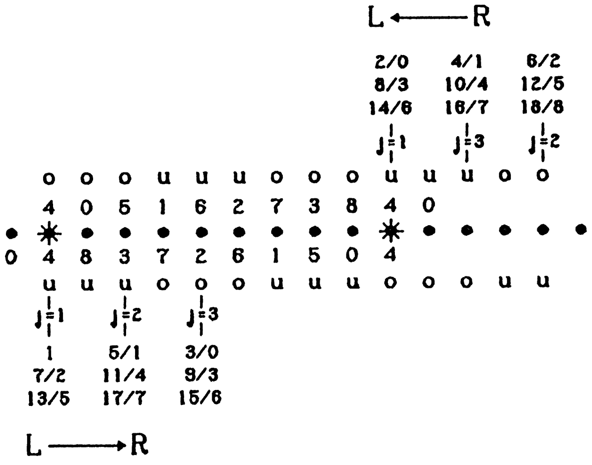
$|p|_r = |11|_3 = 2$ $\triangle h = 4$

$h_{0,j+1} = \left| h_{0,j} + \triangle h \right|_{2r}$ with $1 \leq j \leq r-1$

$h_{0,1} = 1$ for odd half-cycles

$h_{0,1} = 2$ for even half-cycles

$h_{n,j} = h_{0,j} + 2nr$ $\begin{cases} \text{with } n=1,2,3,\ldots \\ \text{and } h_{n,j} \leq 2b \end{cases}$



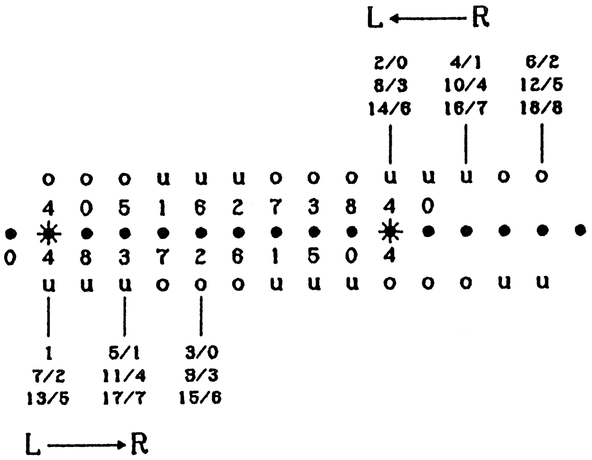
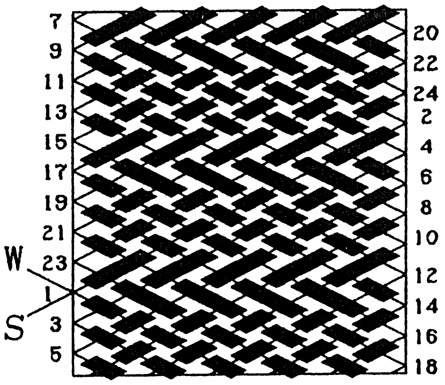
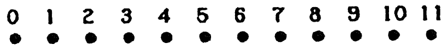


FIG.5





The complementary cyclic bight-number scheme

$|-11|_{12}=1$ $\Delta^{*}=1$

$|p|_r = |11|_4 = 3$ $\Delta h = 6$

$h_{0,j+1} = \left|h_{0,j} + \Delta h\right|_{2r}$ with $1 \leq j \leq r-1$

$h_{0,1} = 1$ for odd half-cycles

$h_{0,1} = 2$ for even half-cycles

$h_{n,j} = h_{0,j} + 2nr$ $\begin{cases} \text{with } n=1,2,3,\ldots \\ \text{and } h_{n,j} \leq 2b \end{cases}$

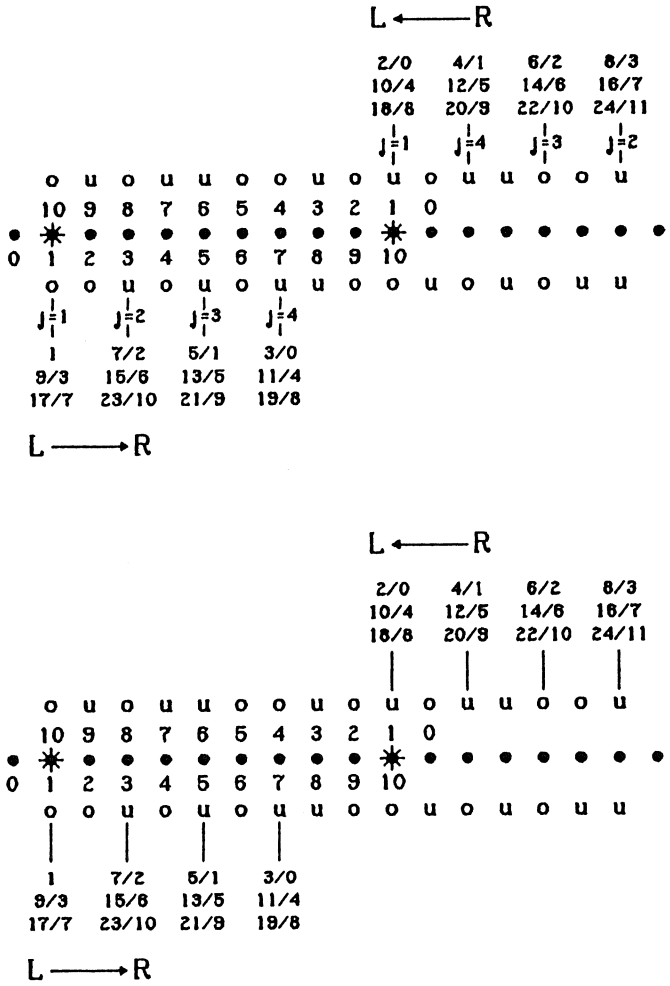
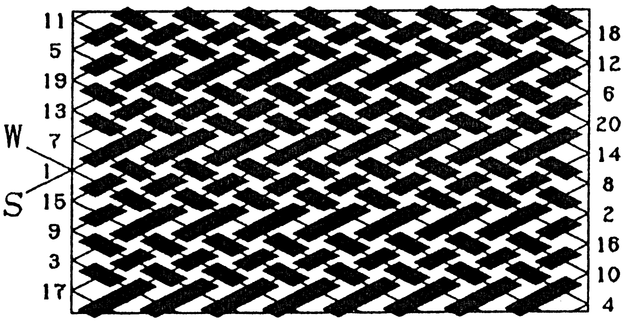


FIG. 6



0 7 4 1 8 5 2 9 6 3

● ● ● ● ● ● ● ● ● ●

The complementary cyclic bight-number scheme

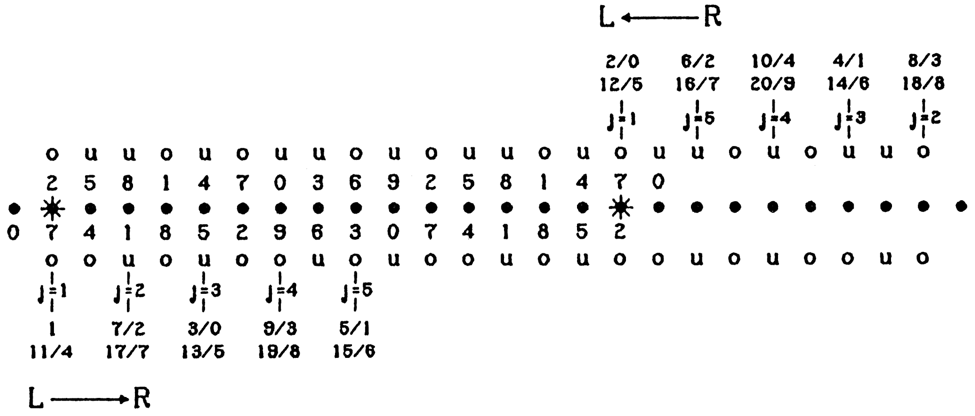
$\left|-17\right|_{10}=3$ $\Delta^{*}=7$

$|p|_r=|17|_5=2$ $\Delta h=6$ $h_{0,j+1}=\left|h_{0,j}+\Delta h\right|_{2r}$ with $1\leq j\leq r-1$

$h_{0,1}=1$ for odd half-cycles

$h_{0,1}=2$ for even half-cycles

$h_{n,j}=h_{0,j}+2nr$ $\begin{cases}\text{with } n=1,2,3,\ldots \\ \text{and } h_{n,j}\leq 2b\end{cases}$



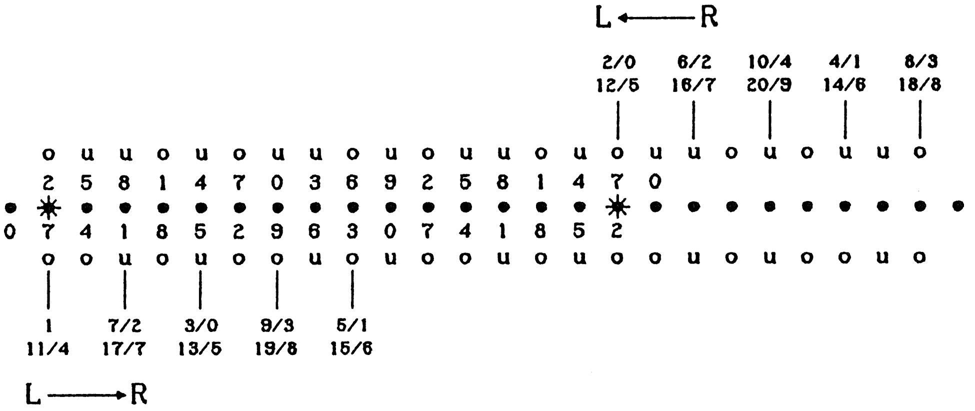


FIG.7

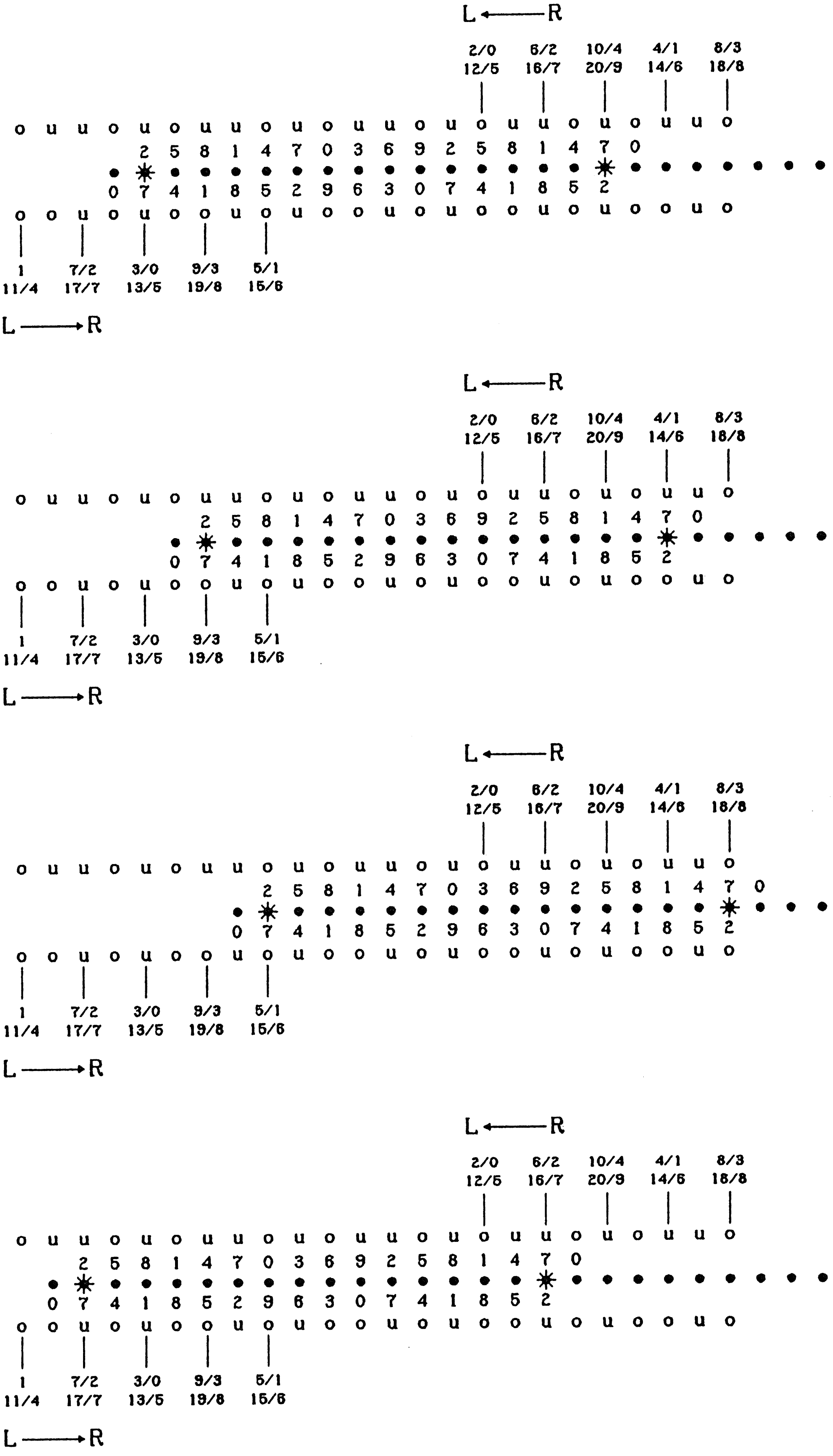
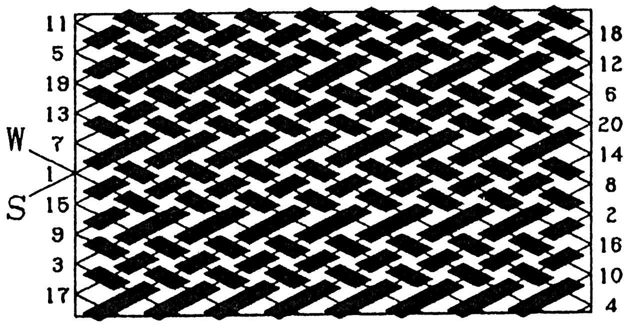

L←R
2/0 12/5
6/2 16/7
10/4 20/9
4/1 14/6
8/3 18/8
o u u o u o u u o u o u u o u o u u o u o u u o
2 5 8 1 4 7 0 3 6 9 2 5 8 1 4 7 0
0 7 4 1 8 5 2 9 6 3 0 7 4 1 8 5 2
o o u o u o o u o u o o u o u o o u o u o o u o
1 11/4
7/2 17/7
3/0 13/5
9/3 19/8
5/1 15/6
L→R
L←R
2/0 12/5
6/2 16/7
10/4 20/9
4/1 14/6
8/3 18/8
o u u o u o u u o u o u u o u o u u o u o u u o
2 5 8 1 4 7 0 3 6 9 2 5 8 1 4 7 0
0 7 4 1 8 5 2 9 6 3 0 7 4 1 8 5 2
o o u o u o o u o u o o u o u o o u o u o o u o
1 11/4
7/2 17/7
3/0 13/5
9/3 19/8
5/1 15/6
L→R
L←R
2/0 12/5
6/2 16/7
10/4 20/9
4/1 14/6
8/3 18/8
o u u o u o u u o u o u u o u o u u o u o u u o
2 5 8 1 4 7 0 3 6 9 2 5 8 1 4 7 0
0 7 4 1 8 5 2 9 6 3 0 7 4 1 8 5 2
o o u o u o o u o u o o u o u o o u o u o o u o
1 11/4
7/2 17/7
3/0 13/5
9/3 19/8
5/1 15/6
L→R
L←R
2/0 12/5
6/2 16/7
10/4 20/9
4/1 14/6
8/3 18/8
o u u o u o u u o u o u u o u o u u o u o u u o
2 5 8 1 4 7 0 3 6 9 2 5 8 1 4 7 0
0 7 4 1 8 5 2 9 6 3 0 7 4 1 8 5 2
o o u o u o o u o u o o u o u o o u o u o o u o
1 11/4
7/2 17/7
3/0 13/5
9/3 19/8
5/1 15/6
L→R

FIG.7A



0 7 4 1 8 5 2 9 6 3

● ● ● ● ● ● ● ● ● ●

The complementary cyclic bight-number scheme

$|-17|_{10}=3$ $\triangle^*=7$

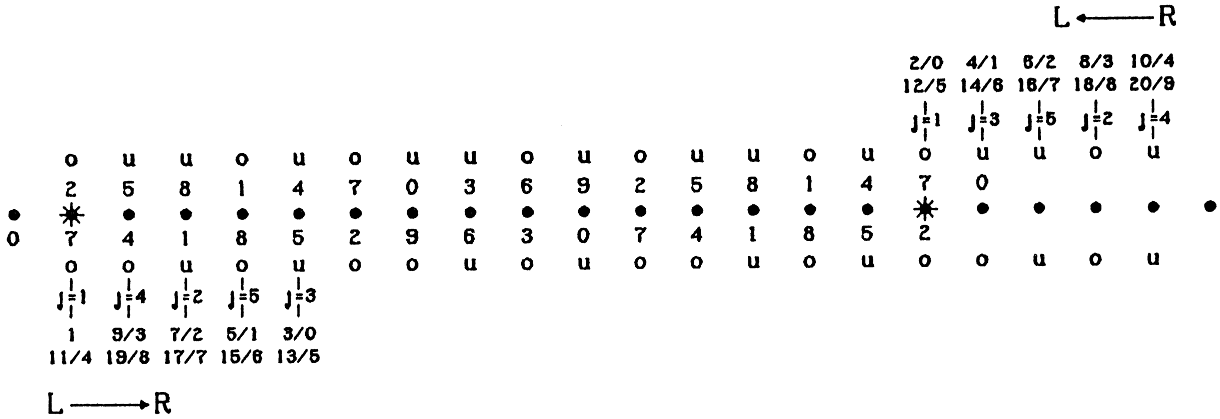
$|p|_r=|17|_5=2$ $\triangle h=6$

$h_{0,j+1}=\left|h_{0,j}+\triangle h\right|_{2r}$ with $1\leq j\leq r-1$

$h_{0,1}=1$ for odd half-cycles

$h_{0,1}=2$ for even half-cycles

$h_{n,j}=h_{0,j}+2nr$ $\begin{cases}\text{with } n=1,2,3,\ldots\\ \text{and } h_{n,j}\leq 2b\end{cases}$



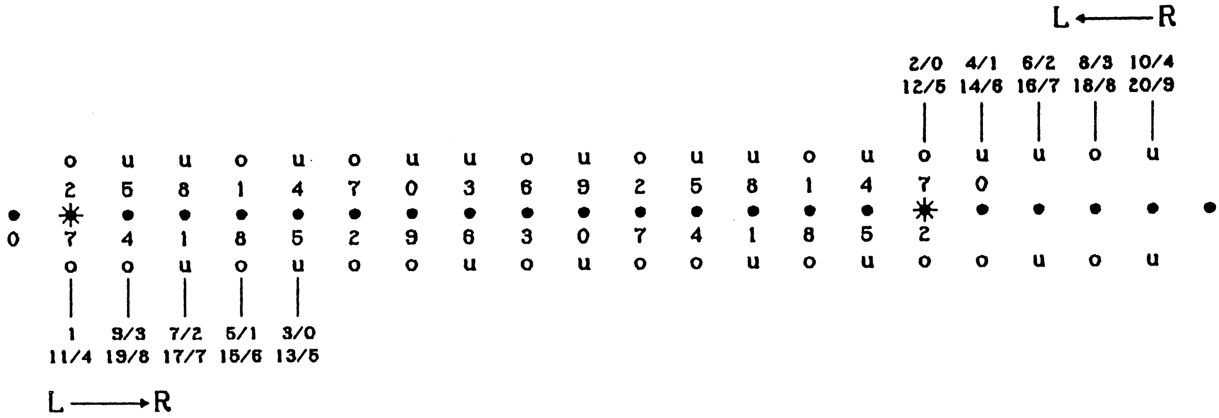


FIG.7B

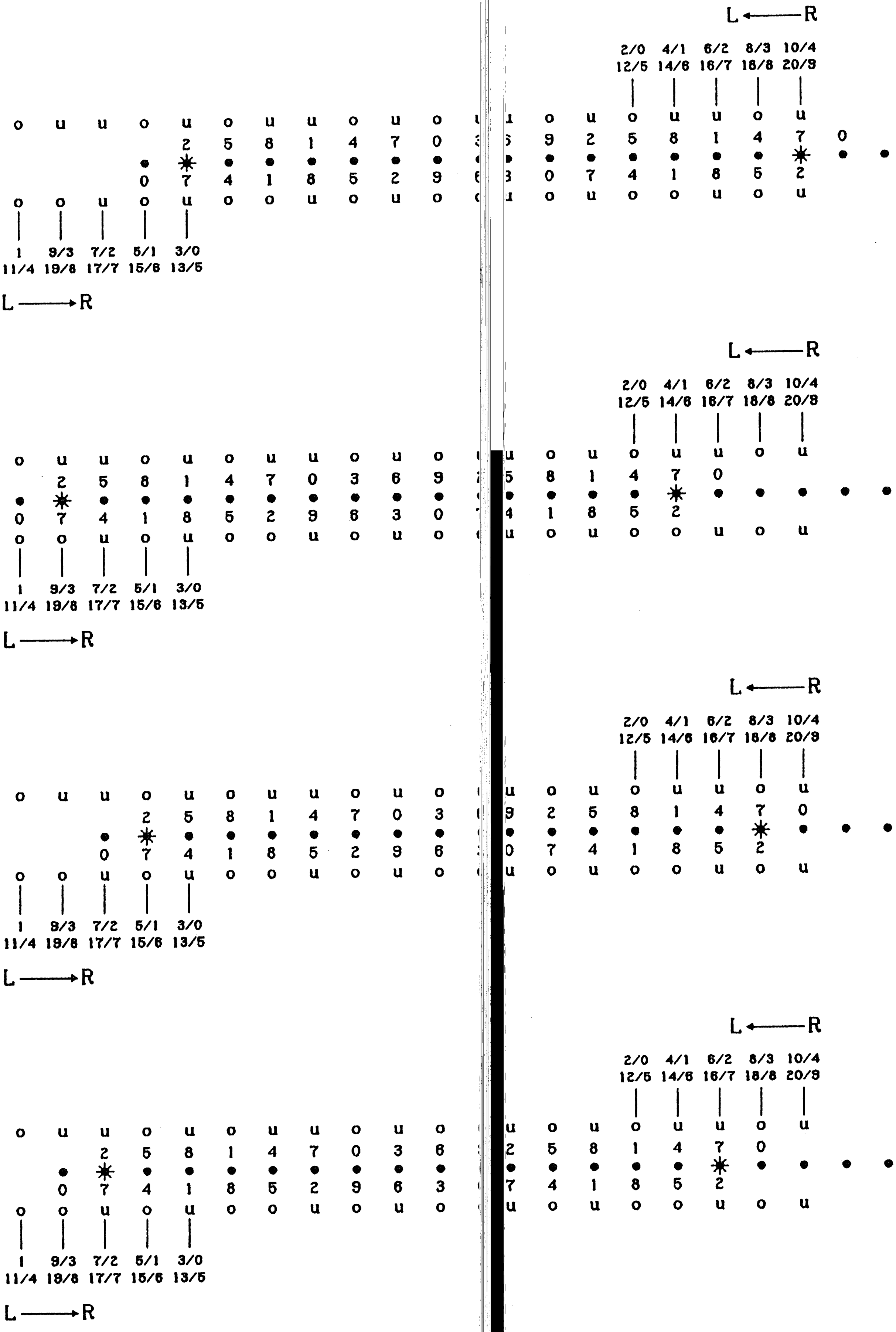

L←R
2/0 4/1 6/2 8/3 10/4
12/5 14/6 16/7 18/8 20/9
o u u o u o u u o u o u o u o u u o u
2 5 8 1 4 7 0 3 6 9 2 5 8 1 4 7 0
0 7 4 1 8 5 2 9 6 3 0 7 4 1 8 5 2
o o u o u o o u o u o u o u o o u o u
1 9/3 7/2 5/1 3/0
11/4 19/8 17/7 15/6 13/5
L→R
L←R
2/0 4/1 6/2 8/3 10/4
12/5 14/6 16/7 18/8 20/9
o u u o u o u u o u o u o u o u u o u
2 5 8 1 4 7 0 3 6 9 5 8 1 4 7 0
0 7 4 1 8 5 2 9 6 3 0 4 1 8 5 2
o o u o u o o u o u o u o u o o u o u
1 9/3 7/2 5/1 3/0
11/4 19/8 17/7 15/6 13/5
L→R
L←R
2/0 4/1 6/2 8/3 10/4
12/5 14/6 16/7 18/8 20/9
o u u o u o u u o u o u o u o u u o u
2 5 8 1 4 7 0 3 9 2 5 8 1 4 7 0
0 7 4 1 8 5 2 9 6 0 7 4 1 8 5 2
o o u o u o o u o u o u o u o o u o u
1 9/3 7/2 5/1 3/0
11/4 19/8 17/7 15/6 13/5
L→R
L←R
2/0 4/1 6/2 8/3 10/4
12/5 14/6 16/7 18/8 20/9
o u u o u o u u o u o u o u o u u o u
2 5 8 1 4 7 0 3 6 2 5 8 1 4 7 0
0 7 4 1 8 5 2 9 6 3 7 4 1 8 5 2
o o u o u o o u o u o u o u o o u o u
1 9/3 7/2 5/1 3/0
11/4 19/8 17/7 15/6 13/5
L→R

FIG.7

Partly Upwards Braiding and Partly Downwards Braiding

The braiding of a knot by starting somewhere near the midpoint of the string-length, which is used in its construction, has some great advantages; especially when the string-length required in its construction is long. It avoids the continuous threading of a long length of string through the braid-work that has so far been completed; thus preventing, or at least minimizing, damage to both. Furthermore, it speeds up the braiding process considerably.

The algorithm-diagrams for Row-coded Regular Knots can readily be adjusted to cater for such desirable braiding procedures.

By starting the braiding of the knot at the midpoint of the string, half the knot is braided "upwards" and half the knot is braided "downwards". Suppose we start with braiding upwards, then the last half-cycle in this upwards braiding-phase should be an *even* half-cycle.

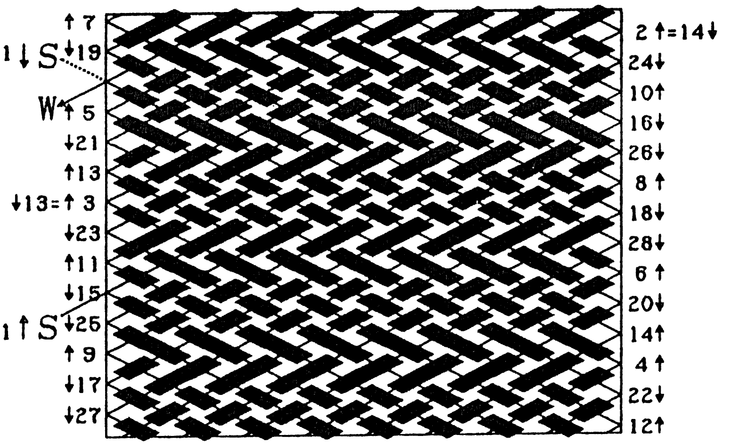
Note that the "Working End" of the end of the last half-cycle in the first phase (the upwards braiding phase) of the braiding process becomes the "Standing End" in the second phase (the downwards braiding phase) of the braiding process.

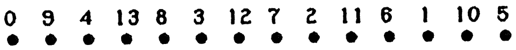
For example, the knot in Fig. 8 with 14 bights has 28 half-cycles in total, and hence the last half-cycle in the, upwards braided, first phase of the construction process should be the 14^{th} half-cycle. From the 15^{th} half-cycle onwards (starting with the other half of the string-length, hence the "Standing End" in the first half of the knot construction becomes the "Working End" in the second half of the knot construction) we braid downwards. Consequently the 2^{nd} half-cycle in the upwards braiding process, becomes the 13^{th} half-cycle in an "overall" downwards braiding process, and the 1^{st} half-cycle in the upwards braiding process, becomes the 14^{th} half-cycle in an "overall" downwards braiding process.

Since we now braid downwards, the Left to Right half-cycle information is set off above the upper coding-sequence in the algorithm-diagram with 13/5 in line with the left-hand star position, and the Right to Left half-cycle information is set off below the lower coding-sequence in the algorithm-diagram with 14/6 in line with the right-hand star position. The half-cycle number/bight-number sets are assigned to the vertical line segments in the same manner as previously. Hence we obtain the two algorithm-diagrams as depicted in Fig. 8.

In Figs. 8*A* and 8*B* are depicted the seven different positions of the sliding central bight-number strip in the algorithm-diagram associated with the half-cycles 1—14 in the upwards braiding process, and in Figs. 8*C* and 8*D* are depicted the seven different positions of the sliding central bight-number strip in the algorithm-diagram associated with the half-cycles 15—28 in the downwards braiding process.

The reader should note that the half-cycle algorithms associated with the half-cycle number/bight-number sets 13/5 and 14/6 in the algorithm-diagram for the downwards braiding phase are **not** used and hence **not** read off. These half-cycle number/bight-number sets are used in this algorithm-diagram as the initial reference sets only!





The complementary cyclic bight-number scheme

$|-17|_{14}=11$ $\Delta^*=9$

$|p|_r=|17|_7=3$ $\Delta h=10$

$h_{0,j+1}=\left|h_{0,j}+\Delta h\right|_{2r}$ with $1\leq j\leq r-1$

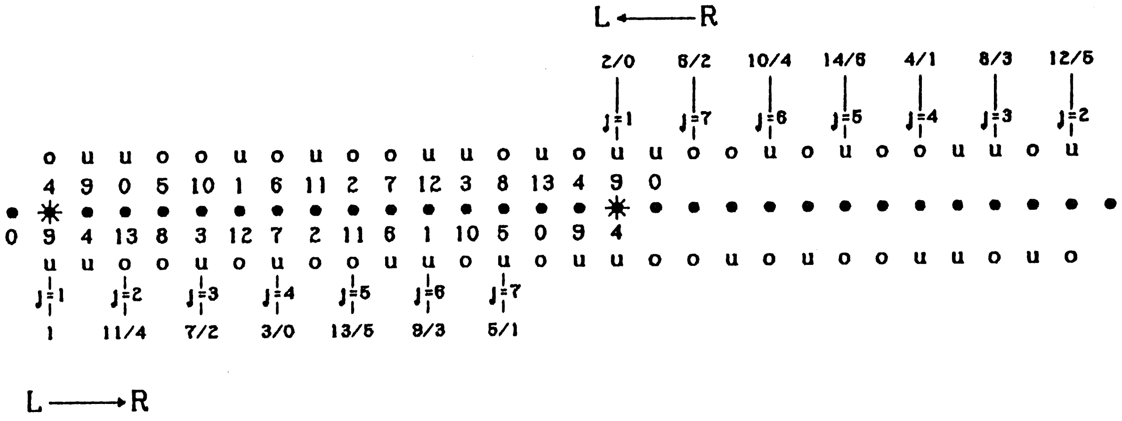
$h_{0,1}=1$ for odd half-cycles

$h_{0,1}=2$ for even half-cycles

half-cycles 1—14 upwards

half-cycles 15—28 downwards

$h_{n,j}=h_{0,j}+2nr$ $\begin{cases}\text{with } n=1,2,3,\dots\\ \text{and } h_{n,j}\leq 2b\end{cases}$


L←R
2/0 6/2 10/4 14/6 4/1 8/3 12/5
j=1 j=7 j=6 j=5 j=4 j=3 j=2
o u u o o u o u o o u u o u o u u o o u o u o o u u o u
4 9 0 5 10 1 6 11 2 7 12 3 8 13 4 9 0
0 9 4 13 8 3 12 7 2 11 6 1 10 5 0 9 4
u u o o u o u o o u u o u o u u o o u o u o o u u o u o
j=1 j=2 j=3 j=4 j=5 j=6 j=7
1 11/4 7/2 3/0 13/5 9/3 5/1
L→R

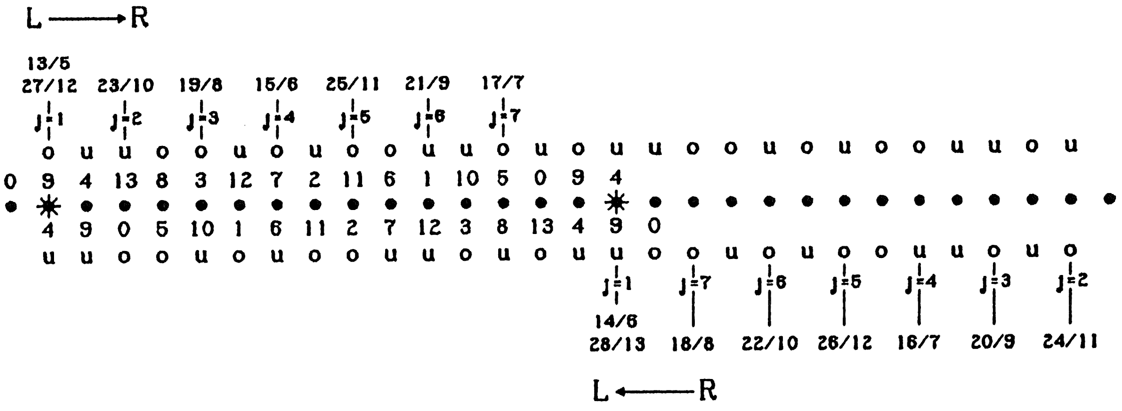

L→R
13/5
27/12 23/10 19/8 15/6 25/11 21/9 17/7
j=1 j=2 j=3 j=4 j=5 j=6 j=7
o u u o o u o u o o u u o u o u u o o u o u o o u u o u
0 9 4 13 8 3 12 7 2 11 6 1 10 5 0 9 4
4 9 0 5 10 1 6 11 2 7 12 3 8 13 4 9 0
u u o o u o u o o u u o u o u u o o u o u o o u u o u o
j=1 j=7 j=6 j=5 j=4 j=3 j=2
14/6
28/13 18/8 22/10 26/12 16/7 20/9 24/11
L←R

FIG. 8

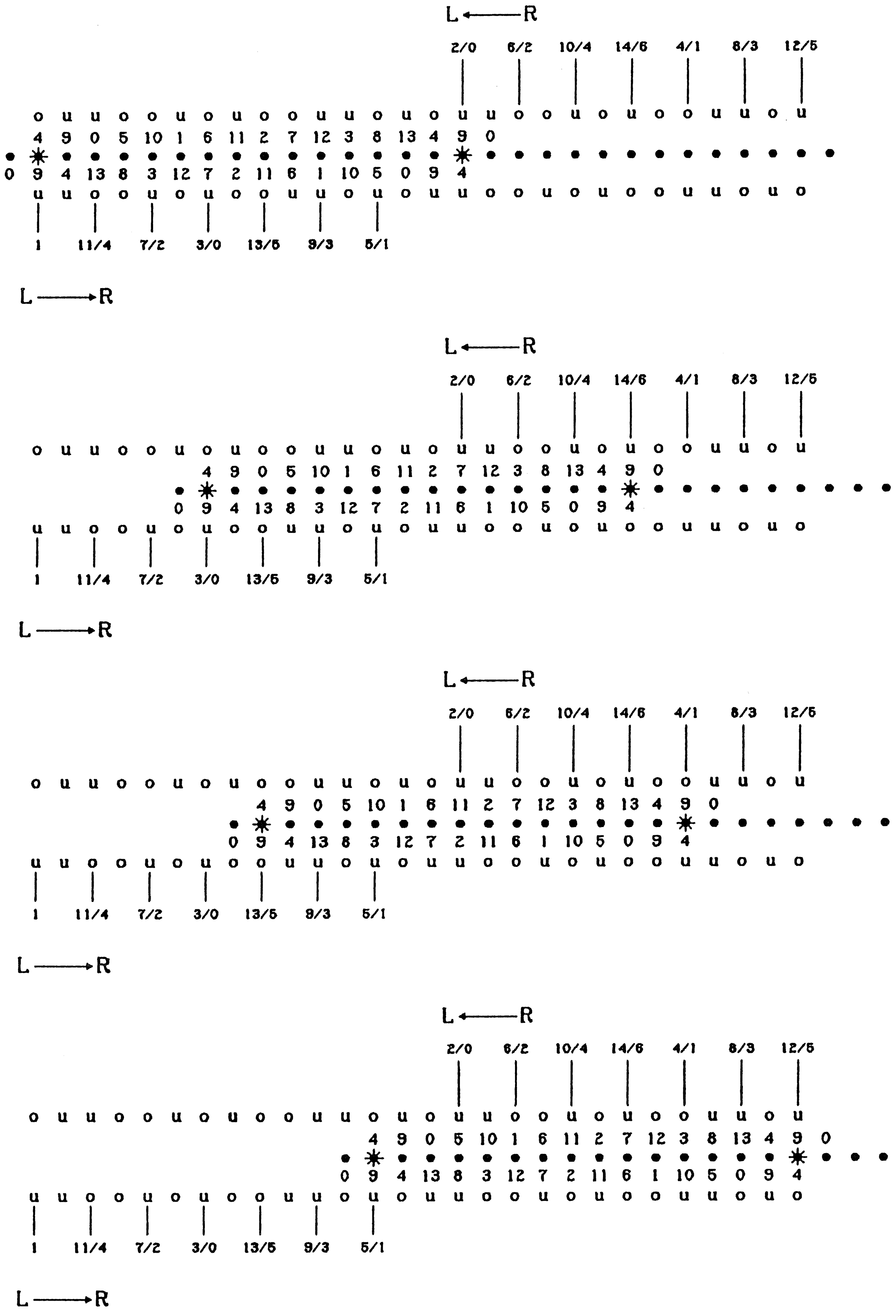


FIG.8A

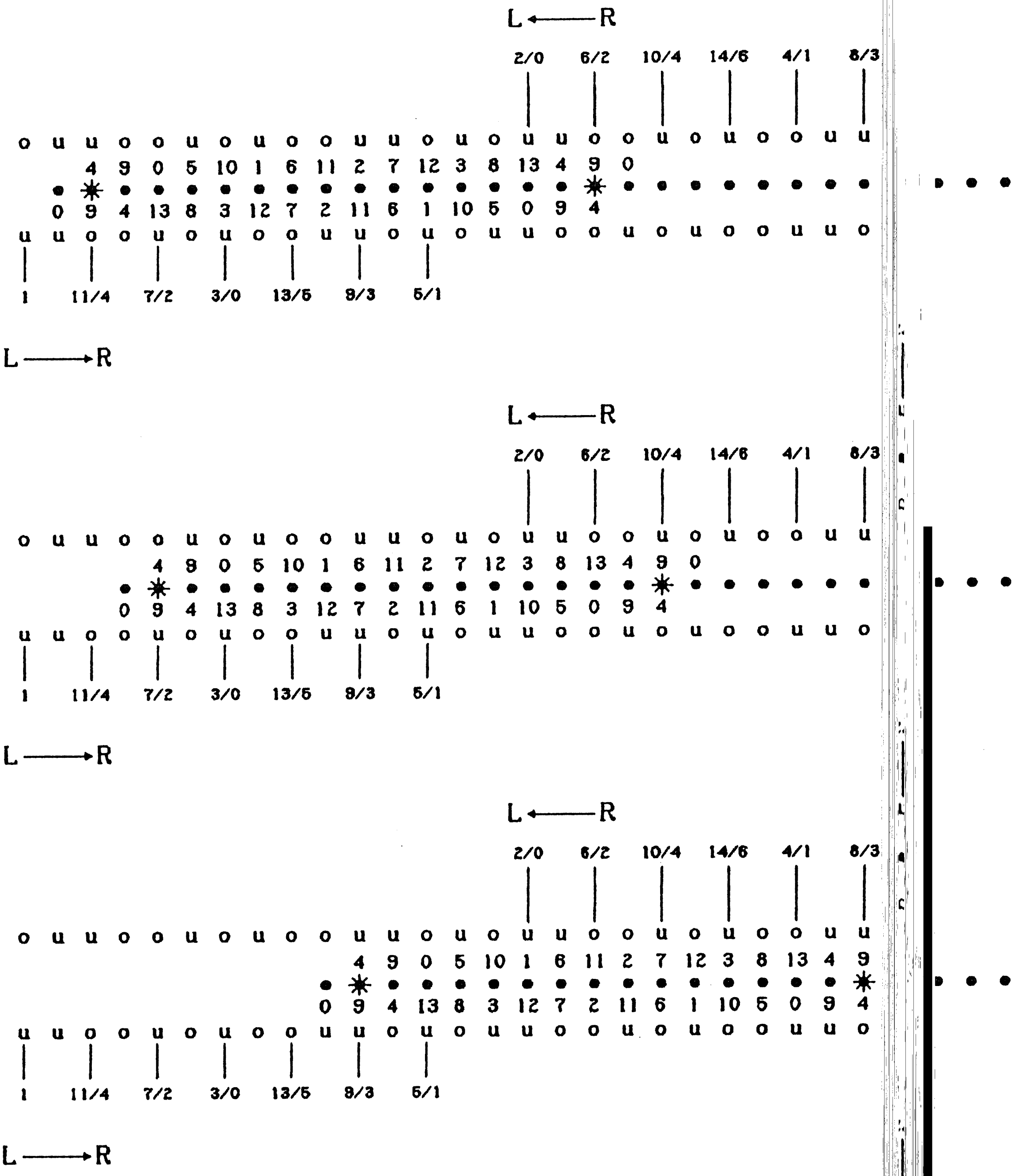

L←R
2/0 6/2 10/4 14/6 4/1 8/3
o u u o o u o u o o u u o u o u u o o u o u o o u u
4 9 0 5 10 1 6 11 2 7 12 3 8 13 4 9 0
0 9 4 13 8 3 12 7 2 11 6 1 10 5 0 9 4
u u o o u o u o o u u o u o u u o o u o u o o u u o
1 11/4 7/2 3/0 13/5 9/3 5/1
L→R
L←R
2/0 6/2 10/4 14/6 4/1 8/3
o u u o o u o u o o u u o u o u u o o u o u o o u u
4 9 0 5 10 1 6 11 2 7 12 3 8 13 4 9 0
0 9 4 13 8 3 12 7 2 11 6 1 10 5 0 9 4
u u o o u o u o o u u o u o u u o o u o u o o u u o
1 11/4 7/2 3/0 13/5 9/3 5/1
L→R
L←R
2/0 6/2 10/4 14/6 4/1 8/3
o u u o o u o u o o u u o u o u u o o u o u o o u u
4 9 0 5 10 1 6 11 2 7 12 3 8 13 4 9
0 9 4 13 8 3 12 7 2 11 6 1 10 5 0 9 4
u u o o u o u o o u u o u o u u o o u o u o o u u o
1 11/4 7/2 3/0 13/5 9/3 5/1
L→R

FIG.8B

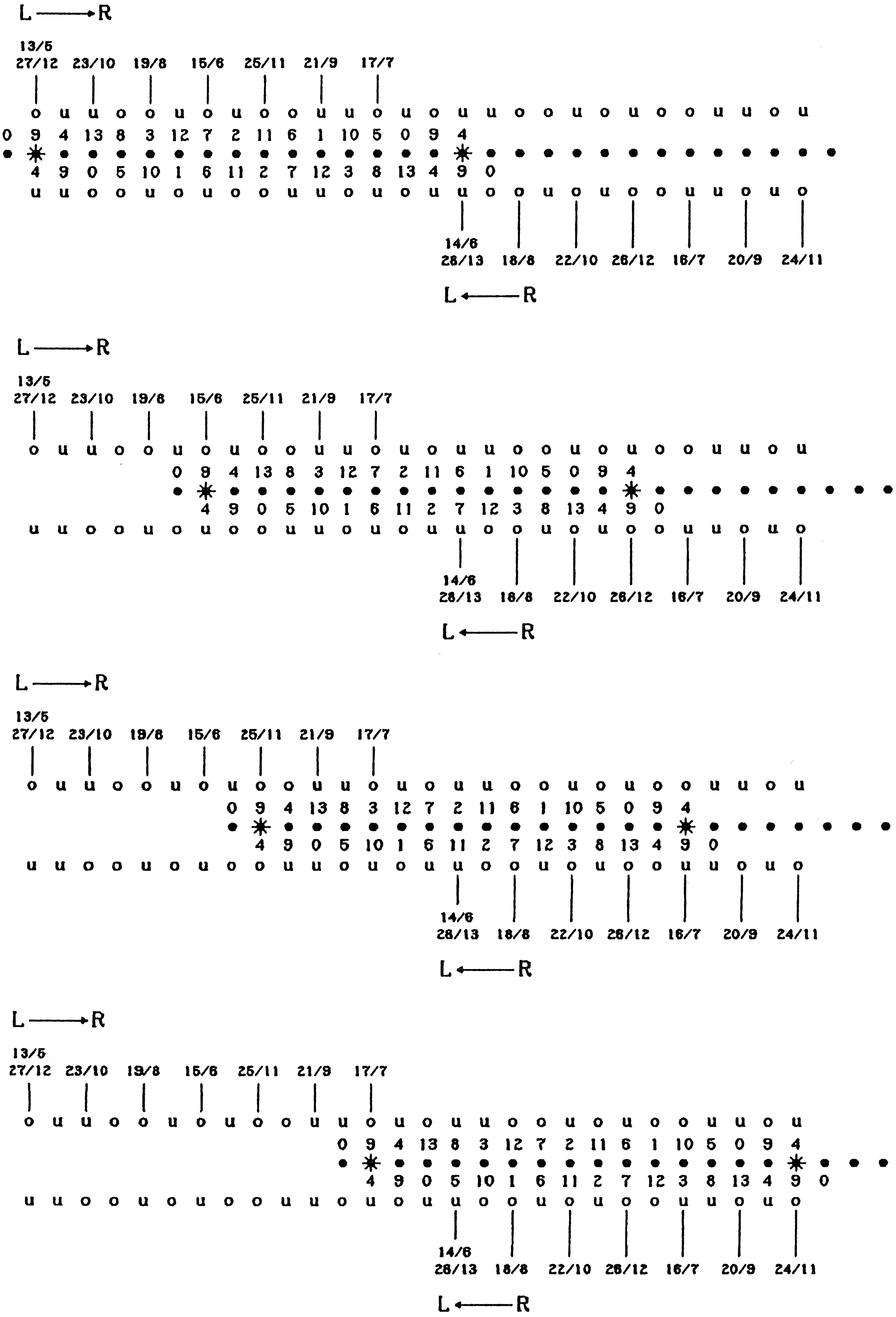

L → R
13/5
27/12 23/10 19/8 15/6 25/11 21/9 17/7
o u u o o u o u o o u u o u o u u o o u o u o o u u o u
0 9 4 13 8 3 12 7 2 11 6 1 10 5 0 9 4
4 9 0 5 10 1 6 11 2 7 12 3 8 13 4 9 0
u u o o u o u o o u u o u o u u o o u o u o o u u o u o
14/6
28/13 18/8 22/10 26/12 16/7 20/9 24/11
L ← R
L → R
13/5
27/12 23/10 19/8 15/6 25/11 21/9 17/7
o u u o o u o u o o u u o u o u u o o u o u o o u u o u
0 9 4 13 8 3 12 7 2 11 6 1 10 5 0 9 4
4 9 0 5 10 1 6 11 2 7 12 3 8 13 4 9 0
u u o o u o u o o u u o u o u u o o u o u o o u u o u o
14/6
28/13 18/8 22/10 26/12 16/7 20/9 24/11
L ← R
L → R
13/5
27/12 23/10 19/8 15/6 25/11 21/9 17/7
o u u o o u o u o o u u o u o u u o o u o u o o u u o u
0 9 4 13 8 3 12 7 2 11 6 1 10 5 0 9 4
4 9 0 5 10 1 6 11 2 7 12 3 8 13 4 9 0
u u o o u o u o o u u o u o u u o o u o u o o u u o u o
14/6
28/13 18/8 22/10 26/12 16/7 20/9 24/11
L ← R
L → R
13/5
27/12 23/10 19/8 15/6 25/11 21/9 17/7
o u u o o u o u o o u u o u o u u o o u o u o o u u o u
0 9 4 13 8 3 12 7 2 11 6 1 10 5 0 9 4
4 9 0 5 10 1 6 11 2 7 12 3 8 13 4 9 0
u u o o u o u o o u u o u o u u o o u o u o o u u o u o
14/6
28/13 18/8 22/10 26/12 16/7 20/9 24/11
L ← R

FIG.8C

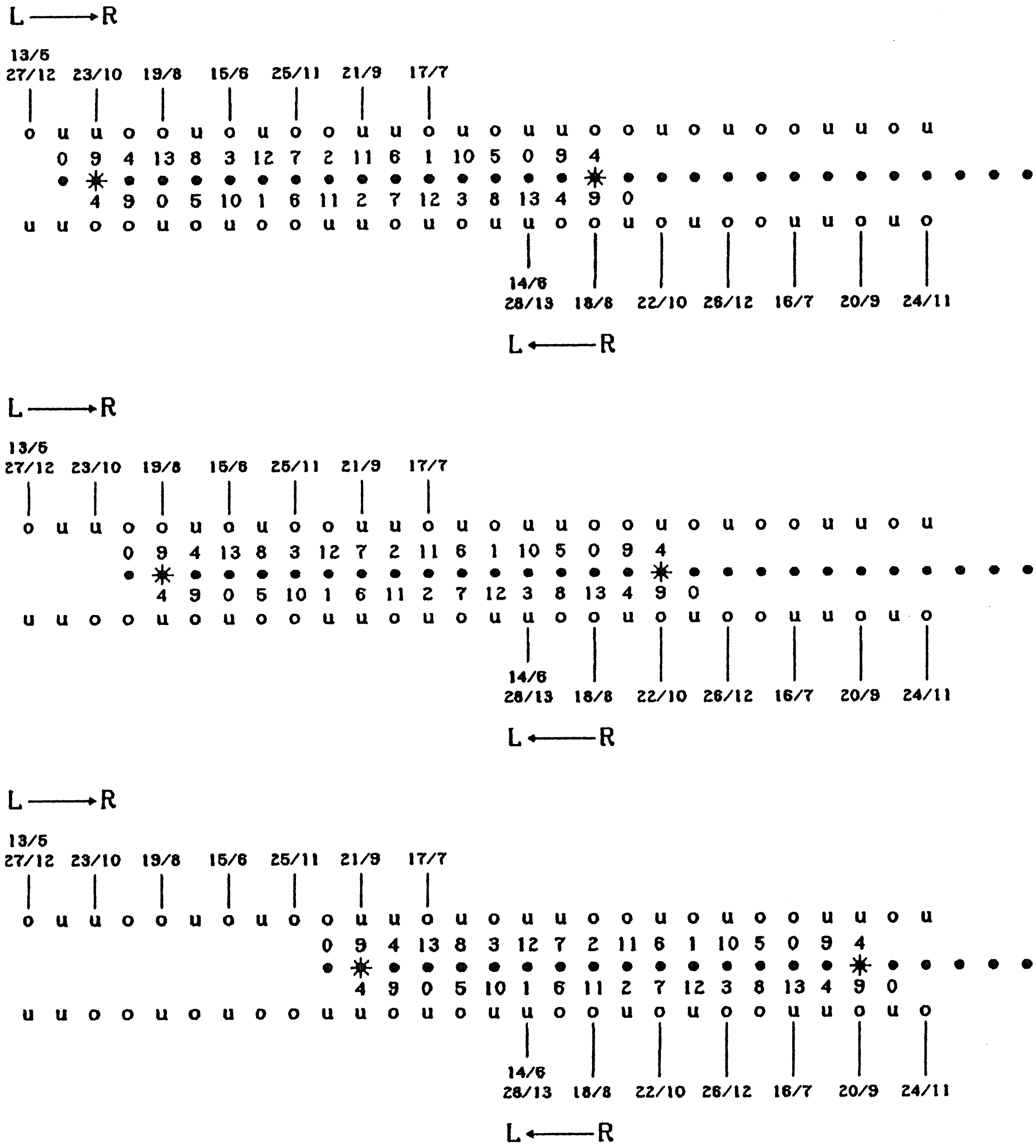

L → R
13/5
27/12 23/10 19/8 15/6 25/11 21/9 17/7
o u u o o u o u o o u u o u o u u o o u o u o o u u o u
0 9 4 13 8 3 12 7 2 11 6 1 10 5 0 9 4
4 9 0 5 10 1 6 11 2 7 12 3 8 13 4 9 0
u u o o u o u o o u u o u o u u o o u o u o o u u o u o
14/6
28/13 18/8 22/10 26/12 16/7 20/9 24/11
L ← R
L → R
13/5
27/12 23/10 19/8 15/6 25/11 21/9 17/7
o u u o o u o u o o u u o u o u u o o u o u o o u u o u
0 9 4 13 8 3 12 7 2 11 6 1 10 5 0 9 4
4 9 0 5 10 1 6 11 2 7 12 3 8 13 4 9 0
u u o o u o u o o u u o u o u u o o u o u o o u u o u o
14/6
28/13 18/8 22/10 26/12 16/7 20/9 24/11
L ← R
L → R
13/5
27/12 23/10 19/8 15/6 25/11 21/9 17/7
o u u o o u o u o o u u o u o u u o o u o u o o u u o u
0 9 4 13 8 3 12 7 2 11 6 1 10 5 0 9 4
4 9 0 5 10 1 6 11 2 7 12 3 8 13 4 9 0
u u o o u o u o o u u o u o u u o o u o u o o u u o u o
14/6
28/13 18/8 22/10 26/12 16/7 20/9 24/11
L ← R

FIG.8D

The half-cycle algorithm-table associated with Figs. 8, $8A$, $8B$, $8C$ and $8D$ is then as follows:

	1.		L $\longrightarrow$ R :	free run .
	2.	$(i=0)$	R $\longrightarrow$ L :	$(s)u$.
	3.	$(i=0)$	L $\longrightarrow$ R :	o .
u	4.	$(i=1)$	R $\longrightarrow$ L :	$(s,1)2u$.
p	5.	$(i=1)$	L $\longrightarrow$ R :	$2o$.
w	6.	$(i=2)$	R $\longrightarrow$ L :	$(s,1)2u-o$.
a	7.	$(i=2)$	L $\longrightarrow$ R :	$o-u-o$.
r	8.	$(i=3)$	R $\longrightarrow$ L :	$(s)u-o-u-o$.
d	9.	$(i=3)$	L $\longrightarrow$ R :	$u-o-2u$.
s	10.	$(i=4)$	R $\longrightarrow$ L :	$(s)o-u-(3,s)4o$.
	11.	$(i=4)$	L $\longrightarrow$ R :	$o-4u-o$.
	12.	$(i=5)$	R $\longrightarrow$ L :	$4o-(s)u-2o$.
	13.	$(i=5)$	L $\longrightarrow$ R :	$5u-o-u$.
	14.	$(i=6)$	R $\longrightarrow$ L :	$(3,s)4o-u-3o$.
	15.	$(i=6)$	L $\longrightarrow$ R :	$5u-o-2u$.
	16.	$(i=7)$	R $\longrightarrow$ L :	$(2,s,1)4o-u-2o-u-o$.
d	17.	$(i=7)$	L $\longrightarrow$ R :	$2u-o-2u-o-3u$.
o	18.	$(i=8)$	R $\longrightarrow$ L :	$o-(s)u-2o-u-3o-u-o$.
w	19.	$(i=8)$	L $\longrightarrow$ R :	$2u-o-3u-o-u-o-u$.
n	20.	$(i=9)$	R $\longrightarrow$ L :	$(s)o-u-3o-u-o-u-o-u-(s)o-u$.
w	21.	$(i=9)$	L $\longrightarrow$ R :	$3u-o-u-o-u-3o-2u$.
a	22.	$(i=10)$	R $\longrightarrow$ L :	$o-u-o-u-o-3u-(s,1)2o-u-o-u$.
r	23.	$(i=10)$	L $\longrightarrow$ R :	$u-o-u-3o-u-o-u-o-2u-o$.
d	24.	$(i=11)$	R $\longrightarrow$ L :	$o-3u-o-u-(s)o-u-2o-2u-o-u$.
s	25.	$(i=11)$	L $\longrightarrow$ R :	$2o-u-2o-2u-2o-u-o-u-2o$.
	26.	$(i=12)$	R $\longrightarrow$ L :	$o-2u-(1,s)2o-2u-o-u-o-2u-2o-u$.
	27.	$(i=12)$	L $\longrightarrow$ R :	$o-u-2o-u-o-u-2o-2u-o-u-o-u$.
	28.	$(i=13)$	R $\longrightarrow$ L :	$2u-(s)o-u-o-2u-2o-u-o-u-2o-2u$.

We don't of course have to braid a knot exactly halfway upwards and halfway downwards, in fact there are sometimes good reasons for deviating more or less from this procedure. For example, it could well be that we want to work the string-ends away at a certain position other than catered for in the current algorithm-diagram, and hence the current half-cycle algorithm-table. Obviously we could change the starting position of the initial (the upwards) braiding phase, but that would involve the construction of a totally new algorithm-diagram and hence the derivation of a completely new associated half-cycle algorithm-table.

It is often possible to make the construction of the algorithm-diagram for the downward braiding-phase simpler by having the upwards and downwards braiding phases slightly unequal. We shall illustrate this in Fig. 9 with the knot discussed above.

We know that the 2^{nd} half-cycle in the **upwards** braiding direction receives in the **downwards** braiding direction the number of the **last odd** half-cycle in the **upwards** braiding phase, and that the 1^{st} half-cycle in the **upwards** braiding direction receives in the **downwards** braiding direction the number of the **last even** half-cycle in the

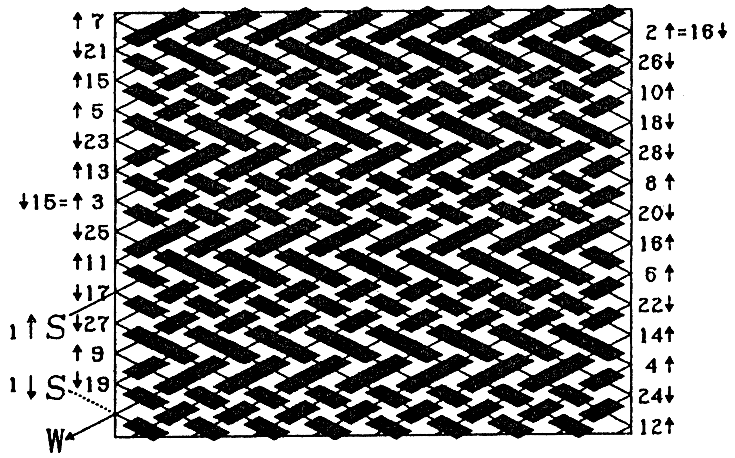
upwards braiding phase. We have seen how we used these properties as refer in the allocation of half-cycle number/bight-number sets for the vertical line segr associated with the downwards braiding phase. It will be evident that when the cycle number/bight-number sets of the last two half-cycles in the **upwards** bra phase coincide with the star positions, we obtain the half-cycle number/bight-nu sets associated with the vertical line segments of the **downwards** braiding phase simple vertical transfer of the sets in the upwards braiding direction which are not in the upwards braiding phase. The reader should compare the algorithm-diagra Fig. 2 with the algorithm-diagrams in Fig. 9. Hence if we braid the Row-coded Re Knot in Fig. 2 partly upwards and partly downwards, then it is convenient to brai half-cycles 1—16 upwards, and the half-cycles 17—28 downwards as is done in Fi

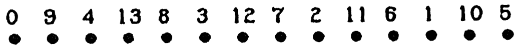
In Figs. 9*A* and 9*B* we have depicted the seven different positions of the sl central bight-number strip in the algorithm-diagram associated with the half-cycle 16 in the upwards braiding process, and in Figs. 8*C* and 8*D* we have depicted the different positions of the sliding central bight-number strip in the algorithm-dia associated with the half-cycles 17—28 in the downwards braiding process.

The half-cycle algorithm-table associated with Figs. 9, 9*A*, 9*B*, 9*C* and 9*D* is th follows:

1. L $\longrightarrow$ R : free run .
2. $(i=0)$ R $\longrightarrow$ L : $(s)u$.
3. $(i=0)$ L $\longrightarrow$ R : o .
4. $(i=1)$ R $\longrightarrow$ L : $(s,1)2u$.
u 5. $(i=1)$ L $\longrightarrow$ R : $2o$.
p 6. $(i=2)$ R $\longrightarrow$ L : $(s,1)2u-o$.
w 7. $(i=2)$ L $\longrightarrow$ R : $o-u-o$.
a 8. $(i=3)$ R $\longrightarrow$ L : $(s)u-o-u-o$.
r 9. $(i=3)$ L $\longrightarrow$ R : $u-o-2u$.
d 10. $(i=4)$ R $\longrightarrow$ L : $(s)o-u-(3,s)4o$.
s 11. $(i=4)$ L $\longrightarrow$ R : $o-4u-o$.
12. $(i=5)$ R $\longrightarrow$ L : $4o-(s)u-2o$.
13. $(i=5)$ L $\longrightarrow$ R : $5u-o-u$.
14. $(i=6)$ R $\longrightarrow$ L : $(3,s)4o-u-3o$.
15. $(i=6)$ L $\longrightarrow$ R : $2u-o-3u-o-u$.
16. $(i=7)$ R $\longrightarrow$ L : $o-u-(s,2)3o-u-o-u-o$.

17. $(i=7)$ L $\longrightarrow$ R : $2u-o-3u-o-2u$.
18. $(i=8)$ R $\longrightarrow$ L : $o-(s)u-3o-u-2o-u-o$.
d 19. $(i=8)$ L $\longrightarrow$ R : $3u-o-2u-o-3u$.
o 20. $(i=9)$ R $\longrightarrow$ L : $(s,1)2o-u-2o-u-3o-u-(s,1)2o$.
w 21. $(i=9)$ L $\longrightarrow$ R : $o-2u-o-3u-o-u-2o-u$.
n 22. $(i=10)$ R $\longrightarrow$ L : $o-u-3o-u-o-(1,s)2u-o-u-o-u$.
w 23. $(i=10)$ L $\longrightarrow$ R : $3u-o-u-o-u-o-u-2o-2u$.
a 24. $(i=11)$ R $\longrightarrow$ L : $o-u-o-u-o-u-(s)o-2u-2o-u-o$
r 25. $(i=11)$ L $\longrightarrow$ R : $u-o-u-3o-2u-o-u-o-2u-o$.
d 26. $(i=12)$ R $\longrightarrow$ L : $o-3u-(s,1)2o-u-o-u-2o-2u-o-u$
s 27. $(i=12)$ L $\longrightarrow$ R : $2o-u-o-u-o-2u-2o-u-o-u-2o$.
28. $(i=13)$ R $\longrightarrow$ L : $o-u-(s)o-u-2o-2u-o-u-o-2u-2$





The complementary cyclic
bight-number scheme

$|-17|_{14}=11$ $\triangle^*=9$

$|p|_r = |17|_7 = 3$ $\triangle h = 10$

$h_{0,j+1} = |h_{0,j} + \triangle h|_{2r}$ with $1 \leq j \leq r-1$

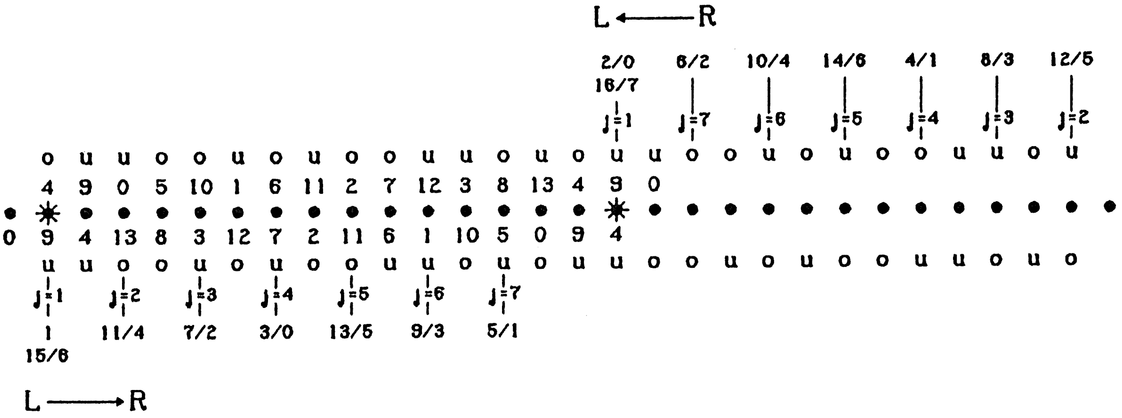
$h_{0,1} = 1$ for odd half-cycles

$h_{0,1} = 2$ for even half-cycles

half-cycles 1—16 upwards

half-cycles 17—28 downwards

$h_{n,j} = h_{0,j} + 2nr$ $\begin{cases} \text{with } n=1,2,3,\ldots \\ \text{and } h_{n,j} \leq 2b \end{cases}$



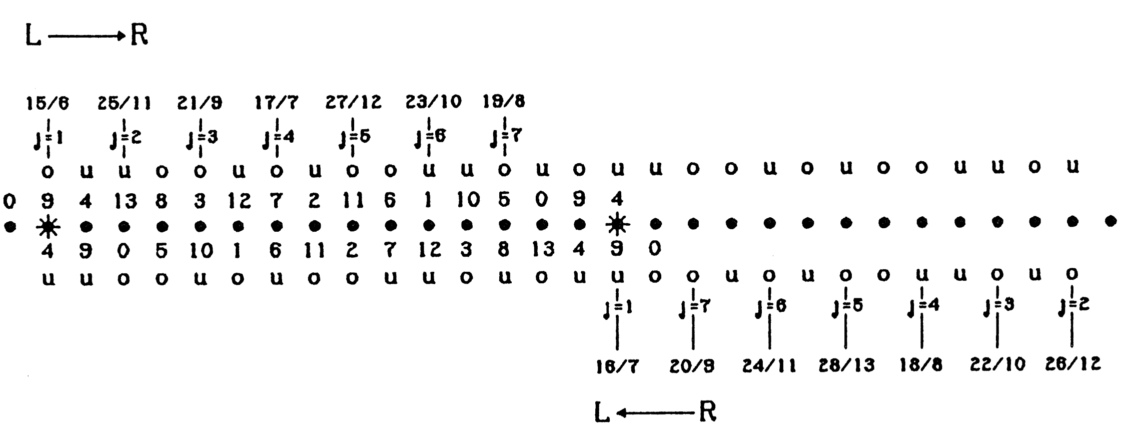


FIG. 9

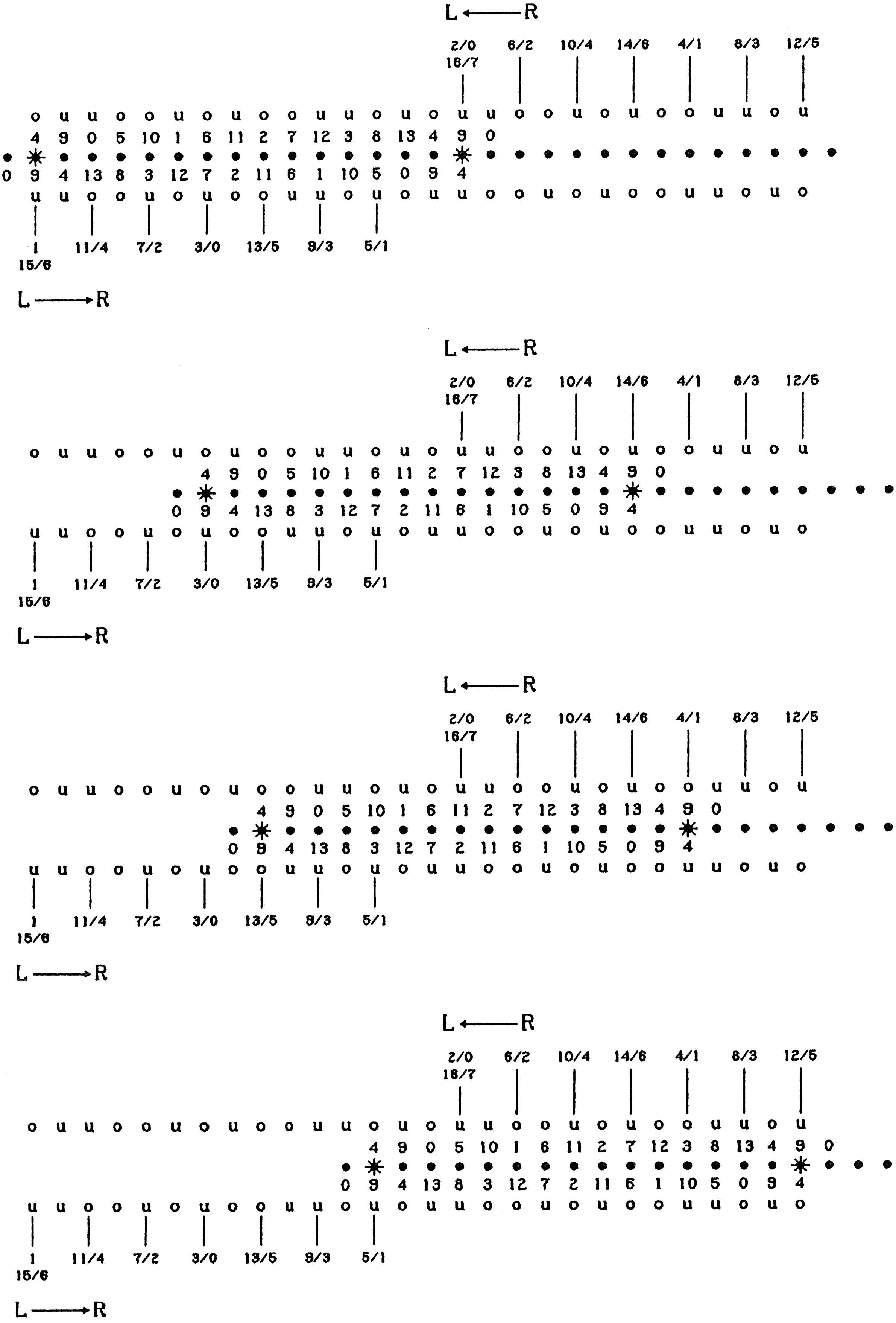

L←R
2/0 6/2 10/4 14/6 4/1 8/3 12/5
16/7
o u u o o u o u o o u u o u o u u o o u o u o o u u o u
4 9 0 5 10 1 6 11 2 7 12 3 8 13 4 9 0
0 9 4 13 8 3 12 7 2 11 6 1 10 5 0 9 4
u u o o u o u o o u u o u o u u o o u o u o o u u o u o
1 11/4 7/2 3/0 13/5 9/3 5/1
15/6
L→R
L←R
2/0 6/2 10/4 14/6 4/1 8/3 12/5
16/7
o u u o o u o u o o u u o u o u u o o u o u o o u u o u
4 9 0 5 10 1 6 11 2 7 12 3 8 13 4 9 0
0 9 4 13 8 3 12 7 2 11 6 1 10 5 0 9 4
u u o o u o u o o u u o u o u u o o u o u o o u u o u o
1 11/4 7/2 3/0 13/5 9/3 5/1
15/6
L→R
L←R
2/0 6/2 10/4 14/6 4/1 8/3 12/5
16/7
o u u o o u o u o o u u o u o u u o o u o u o o u u o u
4 9 0 5 10 1 6 11 2 7 12 3 8 13 4 9 0
0 9 4 13 8 3 12 7 2 11 6 1 10 5 0 9 4
u u o o u o u o o u u o u o u u o o u o u o o u u o u o
1 11/4 7/2 3/0 13/5 9/3 5/1
15/6
L→R
L←R
2/0 6/2 10/4 14/6 4/1 8/3 12/5
16/7
o u u o o u o u o o u u o u o u u o o u o u o o u u o u
4 9 0 5 10 1 6 11 2 7 12 3 8 13 4 9 0
0 9 4 13 8 3 12 7 2 11 6 1 10 5 0 9 4
u u o o u o u o o u u o u o u u o o u o u o o u u o u o
1 11/4 7/2 3/0 13/5 9/3 5/1
15/6
L→R

FIG.9A

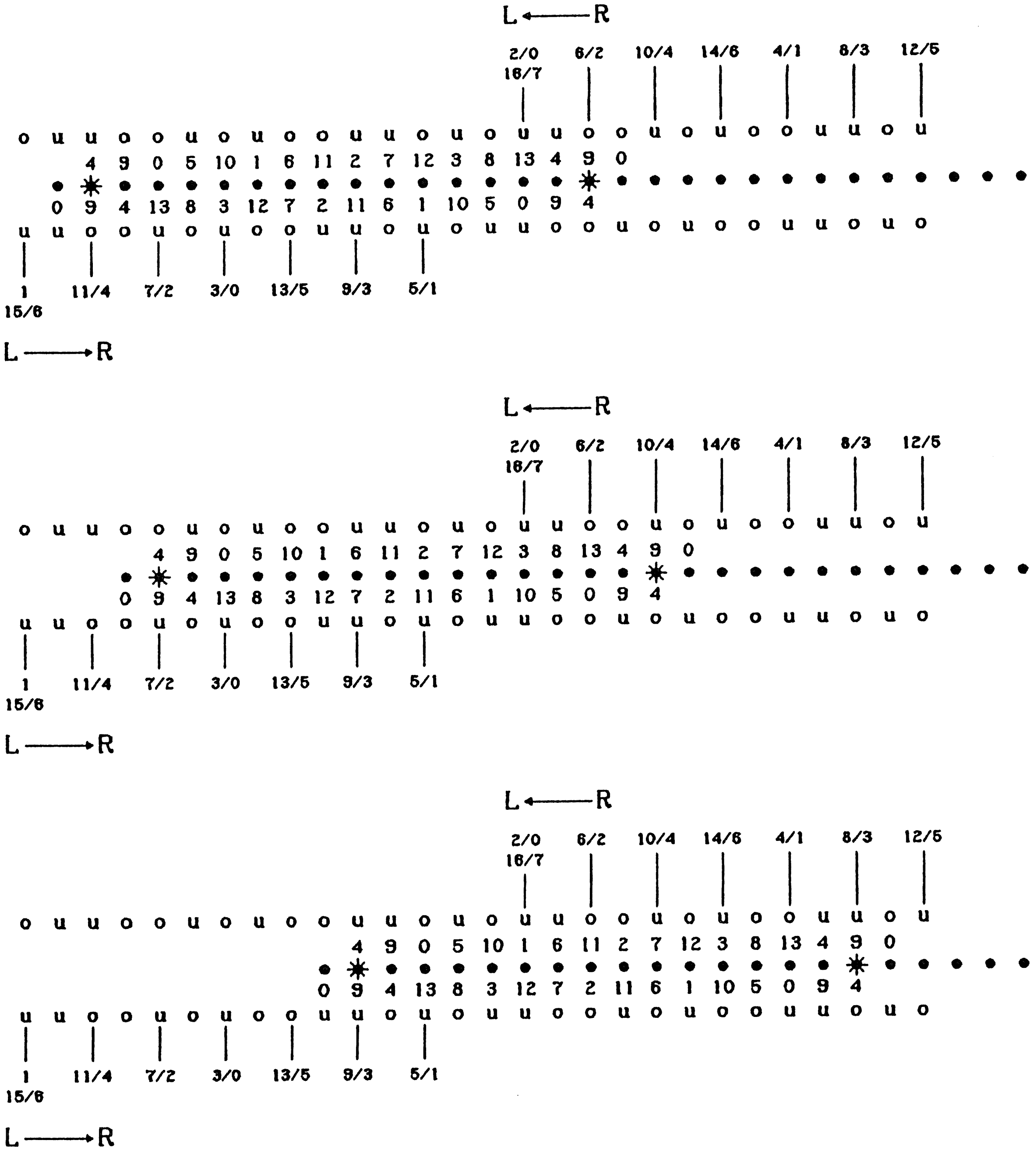

L←R
2/0 16/7
6/2
10/4
14/6
4/1
8/3
12/5
o u u o o u o u o o u u o u o u u o o u o u o o u u o u
4 9 0 5 10 1 6 11 2 7 12 3 8 13 4 9 0
0 9 4 13 8 3 12 7 2 11 6 1 10 5 0 9 4
u u o o u o u o o u u o u o u u o o u o u o o u u o u o
1 15/6
11/4
7/2
3/0
13/5
9/3
5/1
L→R
L←R
2/0 16/7
6/2
10/4
14/6
4/1
8/3
12/5
o u u o o u o u o o u u o u o u u o o u o u o o u u o u
4 9 0 5 10 1 6 11 2 7 12 3 8 13 4 9 0
0 9 4 13 8 3 12 7 2 11 6 1 10 5 0 9 4
u u o o u o u o o u u o u o u u o o u o u o o u u o u o
1 15/6
11/4
7/2
3/0
13/5
9/3
5/1
L→R
L←R
2/0 16/7
6/2
10/4
14/6
4/1
8/3
12/5
o u u o o u o u o o u u o u o u u o o u o u o o u u o u
4 9 0 5 10 1 6 11 2 7 12 3 8 13 4 9 0
0 9 4 13 8 3 12 7 2 11 6 1 10 5 0 9 4
u u o o u o u o o u u o u o u u o o u o u o o u u o u o
1 15/6
11/4
7/2
3/0
13/5
9/3
5/1
L→R

FIG.9B

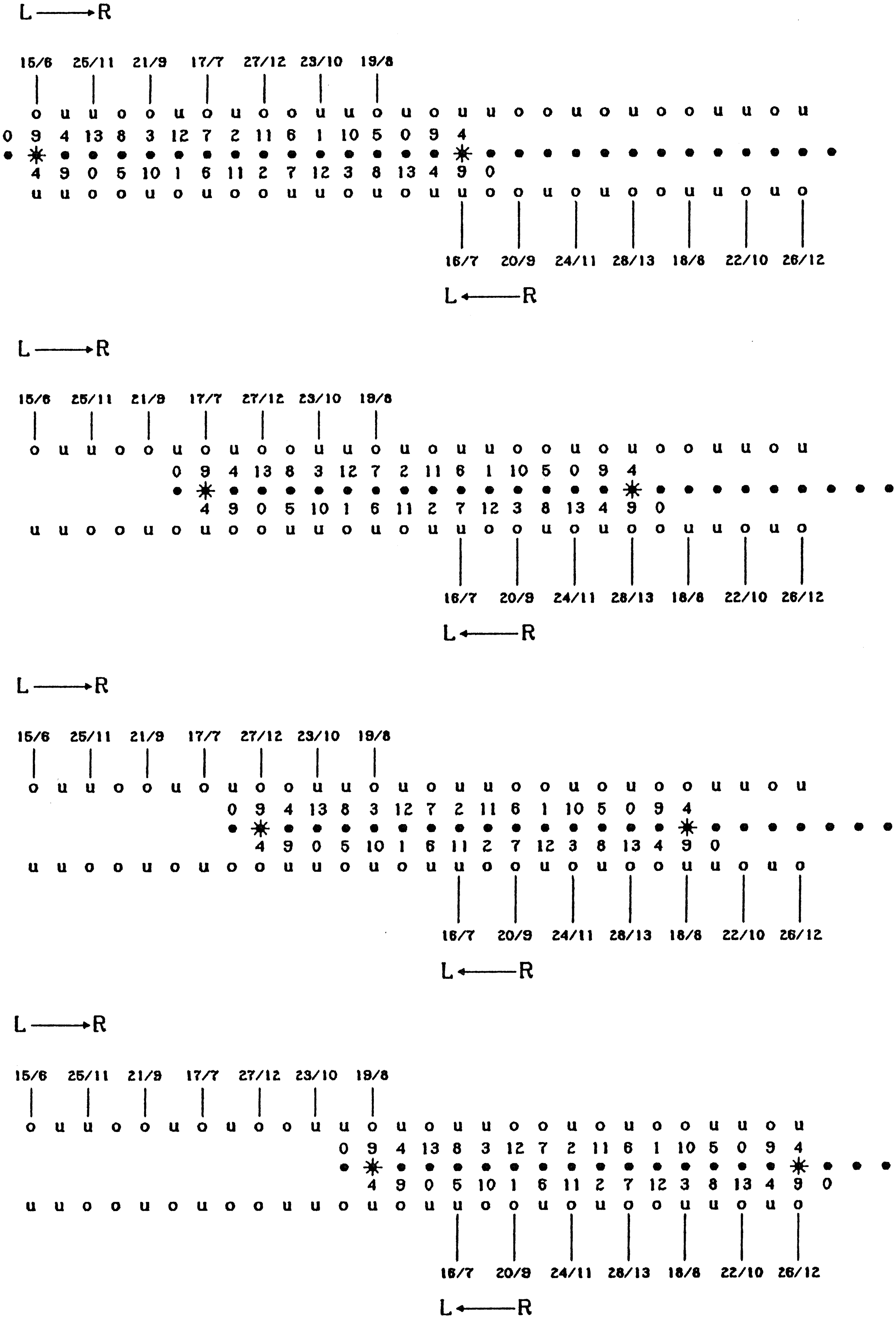

L → R
15/6 25/11 21/9 17/7 27/12 23/10 19/8
o u u o o u o u o o u u o u o u u o o u o u o o u u o u
0 9 4 13 8 3 12 7 2 11 6 1 10 5 0 9 4
4 9 0 5 10 1 6 11 2 7 12 3 8 13 4 9 0
u u o o u o u o o u u o u o u u o o u o u o o u u o u o
16/7 20/9 24/11 28/13 18/8 22/10 26/12
L ← R
L → R
15/6 25/11 21/9 17/7 27/12 23/10 19/8
o u u o o u o u o o u u o u o u u o o u o u o o u u o u
0 9 4 13 8 3 12 7 2 11 6 1 10 5 0 9 4
4 9 0 5 10 1 6 11 2 7 12 3 8 13 4 9 0
u u o o u o u o o u u o u o u u o o u o u o o u u o u o
16/7 20/9 24/11 28/13 18/8 22/10 26/12
L ← R
L → R
15/6 25/11 21/9 17/7 27/12 23/10 19/8
o u u o o u o u o o u u o u o u u o o u o u o o u u o u
0 9 4 13 8 3 12 7 2 11 6 1 10 5 0 9 4
4 9 0 5 10 1 6 11 2 7 12 3 8 13 4 9 0
u u o o u o u o o u u o u o u u o o u o u o o u u o u o
16/7 20/9 24/11 28/13 18/8 22/10 26/12
L ← R
L → R
15/6 25/11 21/9 17/7 27/12 23/10 19/8
o u u o o u o u o o u u o u o u u o o u o u o o u u o u
0 9 4 13 8 3 12 7 2 11 6 1 10 5 0 9 4
4 9 0 5 10 1 6 11 2 7 12 3 8 13 4 9 0
u u o o u o u o o u u o u o u u o o u o u o o u u o u o
16/7 20/9 24/11 28/13 18/8 22/10 26/12
L ← R

FIG. 9C

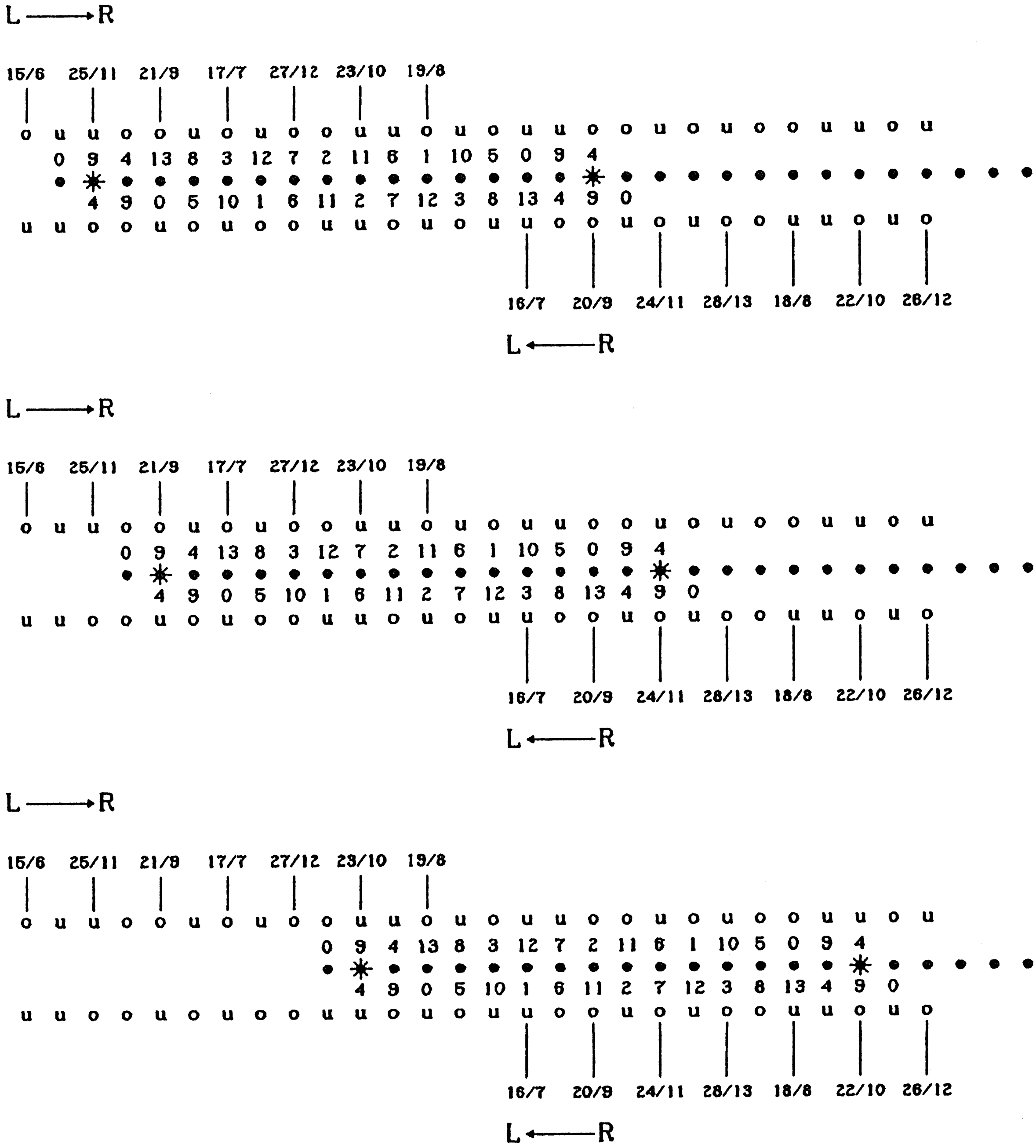

L → R
15/6 25/11 21/9 17/7 27/12 23/10 19/8
o u u o o u o u o o u u o u o u u o o u o u o o u u o u
0 9 4 13 8 3 12 7 2 11 6 1 10 5 0 9 4
4 9 0 5 10 1 6 11 2 7 12 3 8 13 4 9 0
u u o o u o u o o u u o u o u u o o u o u o o u u o u o
16/7 20/9 24/11 28/13 18/8 22/10 26/12
L ← R
L → R
15/6 25/11 21/9 17/7 27/12 23/10 19/8
o u u o o u o u o o u u o u o u u o o u o u o o u u o u
0 9 4 13 8 3 12 7 2 11 6 1 10 5 0 9 4
4 9 0 5 10 1 6 11 2 7 12 3 8 13 4 9 0
u u o o u o u o o u u o u o u u o o u o u o o u u o u o
16/7 20/9 24/11 28/13 18/8 22/10 26/12
L ← R
L → R
15/6 25/11 21/9 17/7 27/12 23/10 19/8
o u u o o u o u o o u u o u o u u o o u o u o o u u o u
0 9 4 13 8 3 12 7 2 11 6 1 10 5 0 9 4
4 9 0 5 10 1 6 11 2 7 12 3 8 13 4 9 0
u u o o u o u o o u u o u o u u o o u o u o o u u o u o
16/7 20/9 24/11 28/13 18/8 22/10 26/12
L ← R

FIG. 9D

The reader should again note that the half-cycle algorithms associated with the half-cycle number/bight-number sets 15/6 and 16/7 in the algorithm-diagram for the downwards braiding phase are **not** used and hence **not** read off. These half-cycle number/bight-number sets are used in this algorithm-diagram as the initial reference sets only!

Although we can transfer vertically the hal -cycle number/bight-number sets of the upwards braiding direction which were not used in the upwards braiding phase, we cannot transfer the associated half-cycle braiding algorithms since the upper coding-sequence in the algorithm-diagram is different to the lower coding-sequence.

The reader will undoubtedly have observed that it is not necessary to draw two separate algorithm-diagrams, one for the upwards braiding phase and one for the downwards braiding phase. It is in fact more convenient to combine the two phases into one algorithm-diagram; an example is shown in Fig. 10. It may well be, however, that the reader finds it more convenient to use the matching central sliding bight-number strip with its associated phase. For this reason we have depicted in Fig. 10*A* the algorithm-diagram with the four positions of the central sliding bight-number strip which is associated with the upwards braiding phase, and in Fig. 10*B* the algorithm-diagram with the four positions of the central sliding bight-number strip which is associated with the downwards braiding phase.

The half-cycle algorithm-table which can now be derived from the algorithm-diagram is as follows:

	1.		L ⟶ R :	free run .
	2.	$(i=0)$	R ⟶ L :	free run .
	3.	$(i=0)$	L ⟶ R :	free run .
u	4.	$(i=1)$	R ⟶ L :	$(s)u$.
p	5.	$(i=1)$	L ⟶ R :	u .
w	6.	$(i=2)$	R ⟶ L :	$o-(s)u$.
a	7.	$(i=2)$	L ⟶ R :	$u-o$.
r	8.	$(i=3)$	R ⟶ L :	$u-(1,s)2o$.
d	9.	$(i=3)$	L ⟶ R :	$2o-u$.
s	10.	$(i=4)$	R ⟶ L :	$u-o-u-$ $s)o$.
	11.	$(i=4)$	L ⟶ R :	$2u-2o$.
	12.	$(i=5)$	R ⟶ L :	$u-o-u-$ $-(s)u$.
	13.	$(i=5)$	L ⟶ R :	$o-u-o-$ $-o$.
	14.	$(i=6)$	R ⟶ L :	$o-u-2o-$ $(1,s)2u$.
d	15.	$(i=6)$	L ⟶ R :	$u-2o-u-$ $o-u$.
o	16.	$(i=7)$	R ⟶ L :	$o-u-o-$ $-2o-(s)u$.
w	17.	$(i=7)$	L ⟶ R :	$o-2u-2o$ $-u-o$.
n	18.	$(i=8)$	R ⟶ L :	$2u-o-u-$ $o-u-(1,s)2o$.
w	19.	$(i=8)$	L ⟶ R :	$o-u-o-$ $u-2o-u$.
a	20.	$(i=9)$	R ⟶ L :	$2o-2u-o$ $-u-o-u-(s)o$.
r	21.	$(i=9)$	L ⟶ R :	$o-u-o-$ $-o-2u-2o$.
d	22.	$(i=10)$	R ⟶ L :	$o-u-2o-$ $2u-o-u-o-(s)u$.
s	23.	$(i=10)$	L ⟶ R :	$u-2o-u-$ $o-u-o-2u-o$.
	24.	$(i=11)$	R ⟶ L :	$o-u-o-$ $-2o-2u-o-u$.

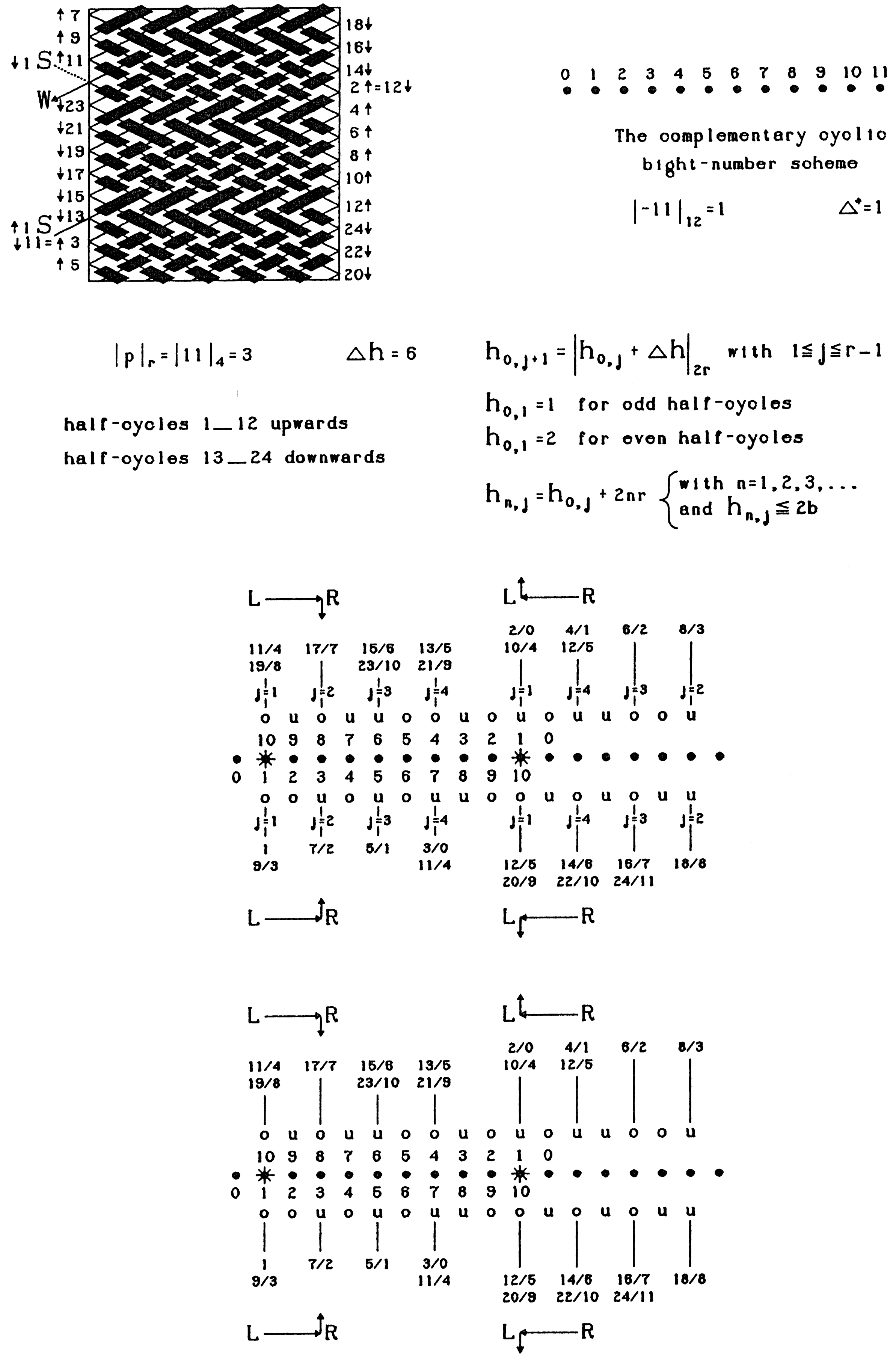

↑7
↑9
↓1 S ↑11
W ↓23
↓21
↓19
↓17
↓15
↓13
↑1 S
↓11=↑3
↑5
18↓
16↓
14↓
2↑=12↓
4↑
6↑
8↑
10↑
12↑
24↓
22↓
20↓
0 1 2 3 4 5 6 7 8 9 10 11
The complementary cyclic bight-number scheme
$|-11|_{12}=1$
$\Delta^{*}=1$
$|p|_r=|11|_4=3$
$\Delta h=6$
$h_{0,j+1}=|h_{0,j}+\Delta h|_{2r}$ with $1\leq j\leq r-1$
$h_{0,1}=1$ for odd half-cycles
$h_{0,1}=2$ for even half-cycles
$h_{n,j}=h_{0,j}+2nr$ with $n=1,2,3,\ldots$ and $h_{n,j}\leq 2b$
half-cycles 1_12 upwards
half-cycles 13_24 downwards
L R
2/0 4/1 6/2 8/3
11/4 17/7 15/6 13/5 10/4 12/5
19/8 23/10 21/9
j=1 j=2 j=3 j=4 j=1 j=4 j=3 j=2
o u o u u o o u o u o u u o o u
10 9 8 7 6 5 4 3 2 1 0
0 1 2 3 4 5 6 7 8 9 10
o o u o u o u u o o u o u o u u
j=1 j=2 j=3 j=4 j=1 j=4 j=3 j=2
1 7/2 5/1 3/0 12/5 14/6 16/7 18/8
9/3 11/4 20/9 22/10 24/11
L R
L R
2/0 4/1 6/2 8/3
11/4 17/7 15/6 13/5 10/4 12/5
19/8 23/10 21/9
o u o u u o o u o u o u u o o u
10 9 8 7 6 5 4 3 2 1 0
0 1 2 3 4 5 6 7 8 9 10
o o u o u o u u o o u o u o u u
1 7/2 5/1 3/0 12/5 14/6 16/7 18/8
9/3 11/4 20/9 22/10 24/11
L R

FIG.10

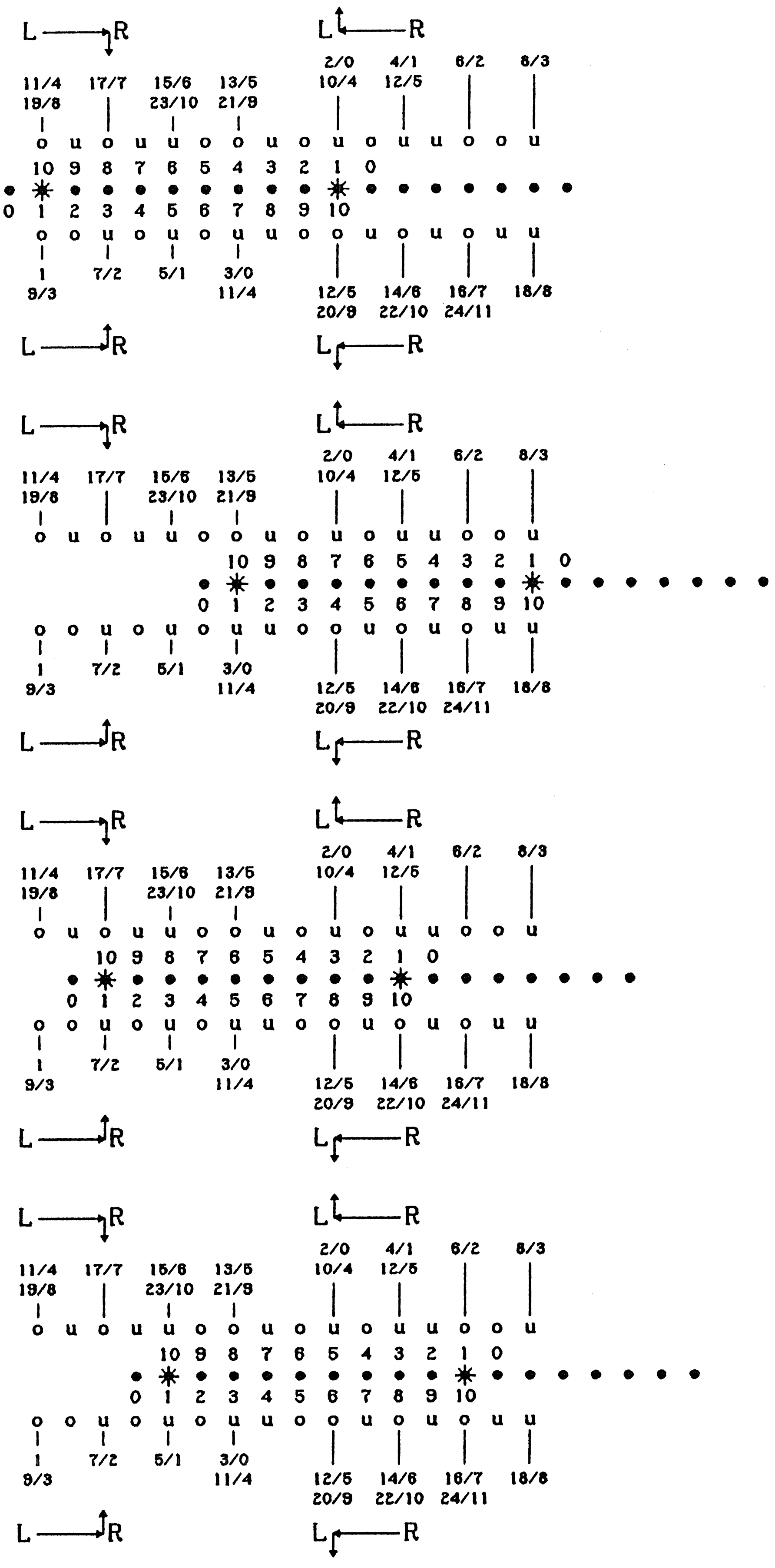

L R
L R
2/0
4/1
6/2
8/3
11/4
19/8
17/7
15/6
23/10
13/5
21/9
10/4
12/5
o u o u u o o u o u o u u o o u
10 9 8 7 6 5 4 3 2 1 0
0 1 2 3 4 5 6 7 8 9 10
o o u o u o u u o o u o u o u u
1
9/3
7/2
5/1
3/0
11/4
12/5
20/9
14/6
22/10
16/7
24/11
18/8
L R
L R
L R
L R
2/0
4/1
6/2
8/3
11/4
19/8
17/7
15/6
23/10
13/5
21/9
10/4
12/5
o u o u u o o u o u o u u o o u
10 9 8 7 6 5 4 3 2 1 0
0 1 2 3 4 5 6 7 8 9 10
o o u o u o u u o o u o u o u u
1
9/3
7/2
5/1
3/0
11/4
12/5
20/9
14/6
22/10
16/7
24/11
18/8
L R
L R
L R
L R
2/0
4/1
6/2
8/3
11/4
19/8
17/7
15/6
23/10
13/5
21/9
10/4
12/5
o u o u u o o u o u o u u o o u
10 9 8 7 6 5 4 3 2 1 0
0 1 2 3 4 5 6 7 8 9 10
o o u o u o u u o o u o u o u u
1
9/3
7/2
5/1
3/0
11/4
12/5
20/9
14/6
22/10
16/7
24/11
18/8
L R
L R
L R
L R
2/0
4/1
6/2
8/3
11/4
19/8
17/7
15/6
23/10
13/5
21/9
10/4
12/5
o u o u u o o u o u o u u o o u
10 9 8 7 6 5 4 3 2 1 0
0 1 2 3 4 5 6 7 8 9 10
o o u o u o u u o o u o u o u u
1
9/3
7/2
5/1
3/0
11/4
12/5
20/9
14/6
22/10
16/7
24/11
18/8
L R
L R

FIG.10A

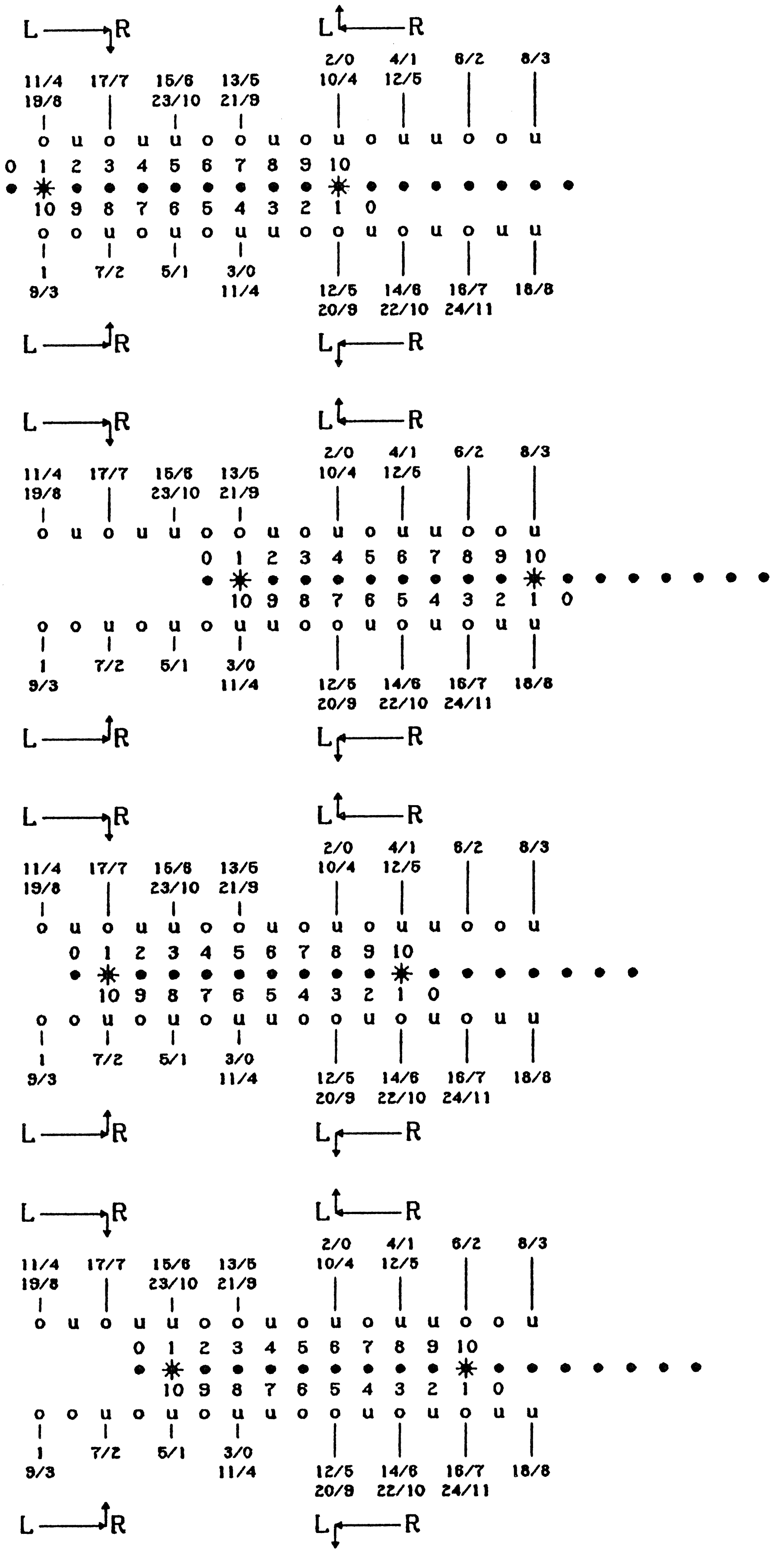

L R
L R
2/0 4/1 6/2 8/3
10/4 12/5
11/4 17/7 15/6 13/5
19/8 23/10 21/9
o u o u u o o u o u o u u o o u
0 1 2 3 4 5 6 7 8 9 10
10 9 8 7 6 5 4 3 2 1 0
o o u o u o u u o o u o u o u u
1 7/2 5/1 3/0
9/3 11/4
12/5 14/6 16/7 18/8
20/9 22/10 24/11
L R
L R
L R
L R
2/0 4/1 6/2 8/3
10/4 12/5
11/4 17/7 15/6 13/5
19/8 23/10 21/9
o u o u u o o u o u o u u o o u
0 1 2 3 4 5 6 7 8 9 10
10 9 8 7 6 5 4 3 2 1 0
o o u o u o u u o o u o u o u u
1 7/2 5/1 3/0
9/3 11/4
12/5 14/6 16/7 18/8
20/9 22/10 24/11
L R
L R
L R
L R
2/0 4/1 6/2 8/3
10/4 12/5
11/4 17/7 15/6 13/5
19/8 23/10 21/9
o u o u u o o u o u o u u o o u
0 1 2 3 4 5 6 7 8 9 10
10 9 8 7 6 5 4 3 2 1 0
o o u o u o u u o o u o u o u u
1 7/2 5/1 3/0
9/3 11/4
12/5 14/6 16/7 18/8
20/9 22/10 24/11
L R
L R
L R
L R
2/0 4/1 6/2 8/3
10/4 12/5
11/4 17/7 15/6 13/5
19/8 23/10 21/9
o u o u u o o u o u o u u o o u
0 1 2 3 4 5 6 7 8 9 10
10 9 8 7 6 5 4 3 2 1 0
o o u o u o u u o o u o u o u u
1 7/2 5/1 3/0
9/3 11/4
12/5 14/6 16/7 18/8
20/9 22/10 24/11
L R
L R

FIG. 10B

A Braiding Start at the Right-hand Bight-boundary

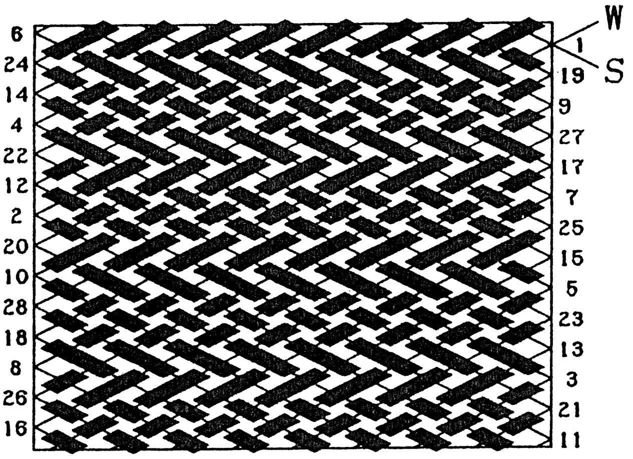
In this Chapter we shall, only very briefly, discuss the required modifications to the algorithm-diagrams associated with a braiding start at the left-hand bight-boundary in order to make them suitable for use in a right-hand bight-boundary braiding start. We shall limit our discussion to the upwards braiding direction, hence the odd-numbered half-cycles run from lower-right to upper-left, and the even-numbered half-cycles run from lower-left to upper-right.

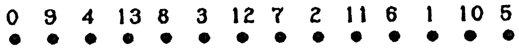
The half-cycle number/bight-number sets above the upper right-hand vertical line segments in an algorithm-diagram for a left-hand bight-boundary braiding start (upwards braiding method) must, for an algorithm-diagram associated with a right-hand bight-boundary start (upwards braiding method), be modified by **decreasing** the values of both the half-cycle numbers and the bight-numbers by 1.

The half-cycle number/bight-number sets below the lower left-hand vertical line segments in an algorithm-diagram for a left-hand bight-boundary braiding start (upwards braiding method) must, for an algorithm-diagram associated with a right-hand bight-boundary start (upwards braiding method), be modified by **decreasing** only the values of the half-cycle numbers by 1 (the old half-cycle number 1 will then be modified into 0, which is half-cycle number $2b$ with associated bight-number $(b-1)$).

We can, of course, also by means of calculation determine the various modified values of the half-cycle number/bight-number sets. The same formulae as used for a left-hand bight-boundary braiding start are valid, except that now $h_{0,1}/i_{0,1} = 2r/(r-1)$ for even half-cycles.

Fig. 11 illustrates an example in which the algorithm-diagram of Fig. 2 has been modified for a right-hand bight-boundary braiding start (upwards braiding direction).





The complementary cyclic bight-number scheme

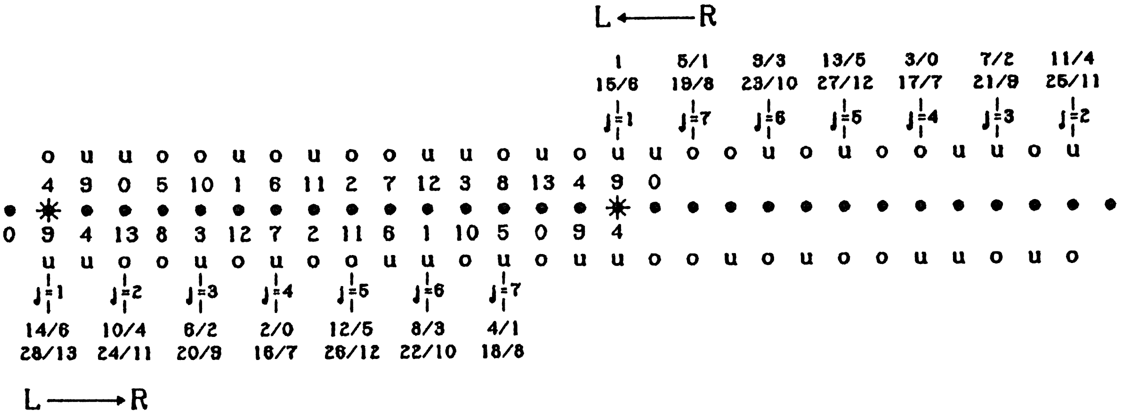
$|-17|_{14} = 11$ $\triangle^* = 9$

$|p|_r = |17|_7 = 3$ $\triangle h = 10$ $h_{0,j+1} = \left|h_{0,j} + \triangle h\right|_{2r}$ with $1 \leq j \leq r-1$

$h_{0,1} = 1$ for odd half-cycles

$h_{0,1} = 2r$ for even half-cycles

$h_{n,j} = h_{0,j} + 2nr \begin{cases} \text{with } n = 1, 2, 3, \ldots \\ \text{and } h_{n,j} \leq 2b \end{cases}$



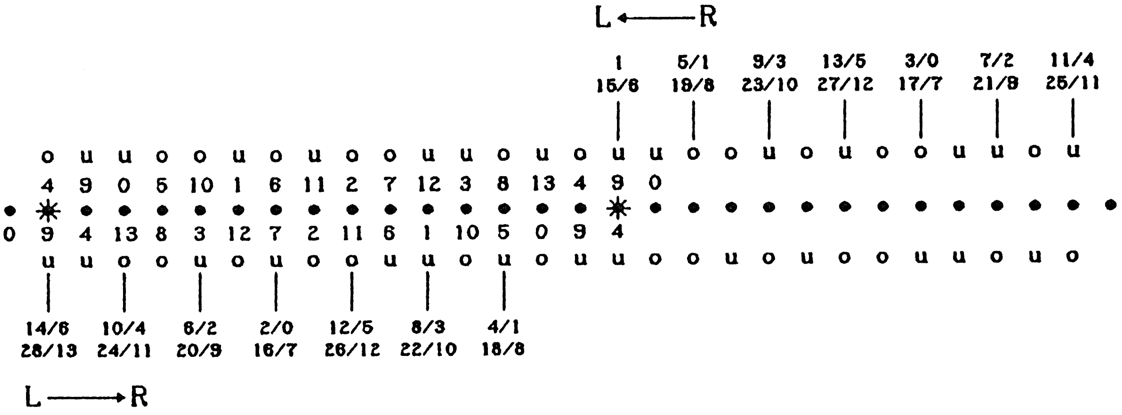


FIG.11

Finally the following half-cycle algorithm-table can be read off from this algorithm-diagram:

1. $\mathrm{R} \longrightarrow \mathrm{L}$: free run .
2. $(i=0)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $(s)o$.
3. $(i=0)$ $\mathrm{R} \longrightarrow \mathrm{L}$: u .
4. $(i=1)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $(s,1)2o$.
5. $(i=1)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $u-o$.
6. $(i=2)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $(s)o-u-o$.
7. $(i=2)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $o-u-o$.
8. $(i=3)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $(s)u-o-2u$.
9. $(i=3)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $u-3o$.
10. $(i=4)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $(s)o-4u-(s)o$.
11. $(i=4)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $6o$.
12. $(i=5)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $(4,s)5u-o-u$.
13. $(i=5)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $3o-u-3o$.
14. $(i=6)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $2u-o-(s,2)3u-o-u$.
15. $(i=6)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $o-u-2o-u-o-u-o$.
16. $(i=7)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $o-(1,s)2u-o-u-o-u-2o$.
17. $(i=7)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $o-u-o-u-o-3u-o$.
18. $(i=8)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $o-(s)u-o-u-3o-u-2o$.
19. $(i=8)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $u-o-3u-o-2u-o-u$.
20. $(i=9)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $(s)u-3o-u-2o-u-2o-(s)u-o$.
21. $(i=9)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $2u-o-2u-o-3u-o-2u$.
22. $(i=10)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $u-2o-u-3o-u-(s,1)2o-2u-o$.
23. $(i=10)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $u-o-3u-o-u-2o-u-o-u-o$.
24. $(i=11)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $3o-u-o-(1,s)2u-o-u-o-2u-2o$.
25. $(i=11)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $u-o-u-o-u-o-u-2o-2u-o-u-o$.
26. $(i=12)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $o-u-o-u-(s)o-2u-2o-u-o-u-2o-u$.
27. $(i=12)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $u-3o-2u-o-u-o-2u-2o-u-o$.
28. $(i=13)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $2u-(s,1)2o-u-o-u-2o-2u-o-u-o-2u$.

REFERENCES

1. Braiding — Regular Knots (1988).
   A.G.Schaake, J.C.Turner and D.A.Sedgwick; Department of Mathematics and Statistics, University of Waikato, Private Bag 3105, Hamilton, New Zealand.

2. A New Theory of Braiding: Research Report 1/2 (1989).
   A.G.Schaake, and J.C.Turner; Department of Mathematics and Statistics, University of Waikato, Private Bag 3105, Hamilton, New Zealand.

3. The Braiding of Column-Coded Regular Knots; Pamphlet No.7 (1992).
   A.G.Schaake, and J.C.Turner; Australian Edition: The Australian Plaiters and Whipmakers Association, Rams Skull Press, Mr. Ron Edwards, 12 Fairyland Road, Kuranda Qld 4872, Australia. English Edition: Footrope Knots, Des & Liz Pawson, 501 Wherstead Road, Ipswich, Suffolk IP2 8LL, England.

Copies of this pamphlet, and items 1, 2 *and* 3, *may be obtained from:*

The Australian Plaiters and Whipmakers Association, Rams Skull Press, Mr. Ron Edwards, 12 Fairyland Road, Kuranda Qld 4872, Australia.

Footrope Knots, Des & Liz Pawson, 501 Wherstead Road, Ipswich, Suffolk IP2 8LL, England.

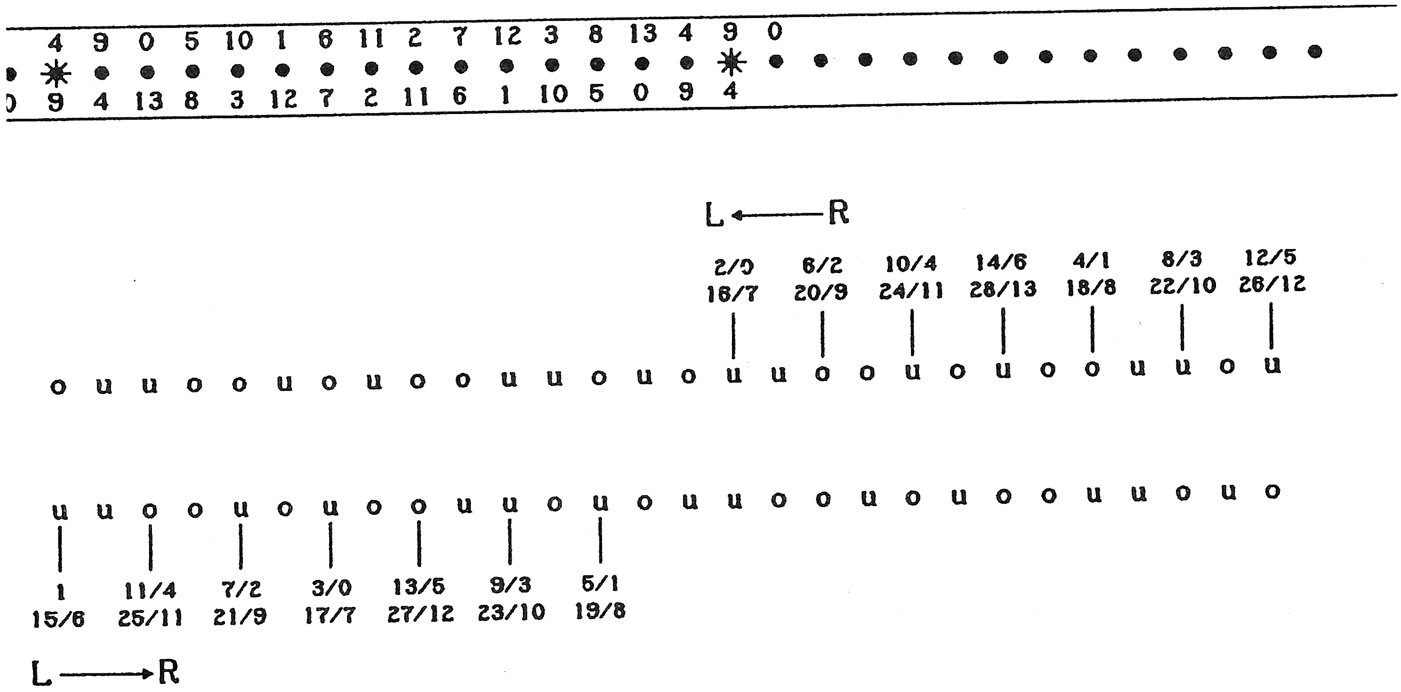

4 9 0 5 10 1 6 11 2 7 12 3 8 13 4 9 0
0 9 4 13 8 3 12 7 2 11 6 1 10 5 0 9 4
L ← R
2/0 16/7
6/2 20/9
10/4 24/11
14/6 28/13
4/1 18/8
8/3 22/10
12/5 26/12
o u u o o u o u o o u u o u o u u o o u o u o o u u o u
u u o o u o u o o u u o u o u u o o u o u o o u u o u o
1 15/6
11/4 25/11
7/2 21/9
3/0 17/7
13/5 27/12
9/3 23/10
5/1 19/8
L → R

FIG.2

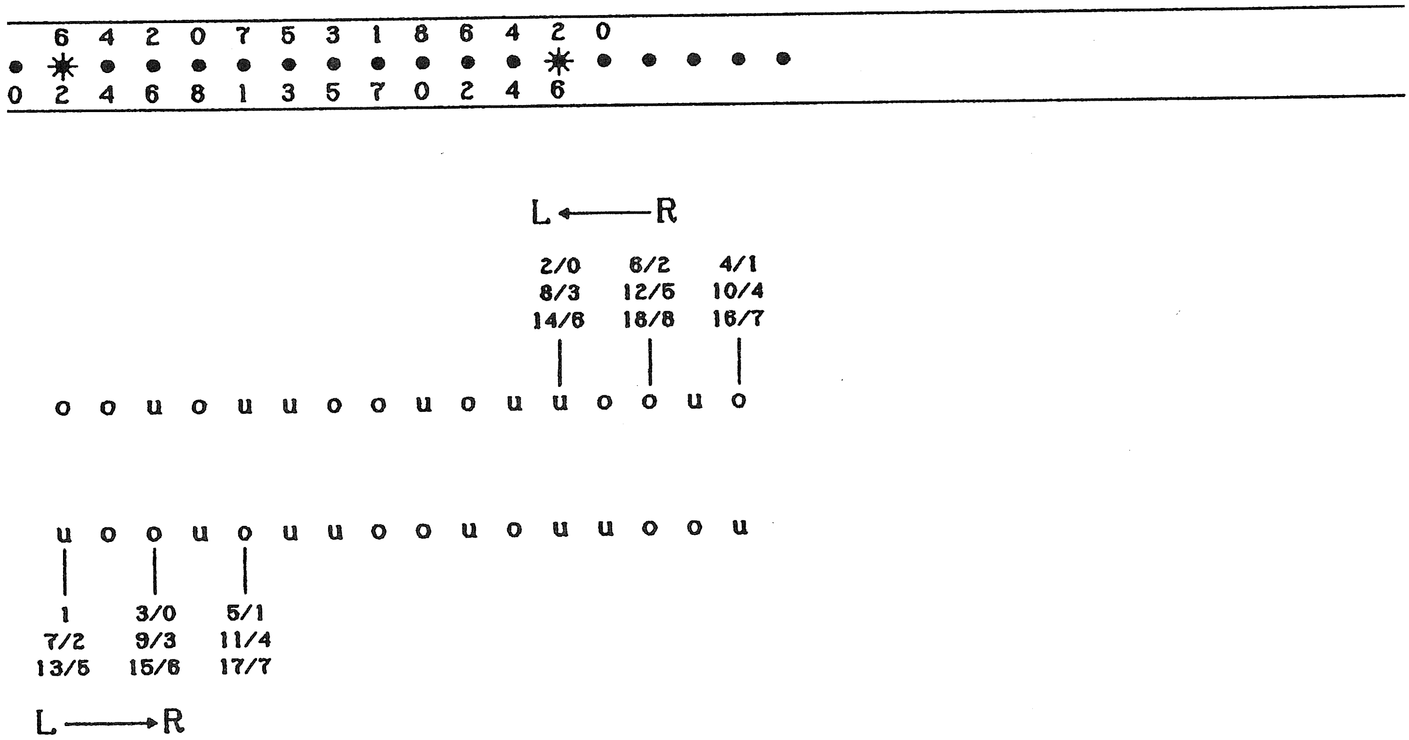

6 4 2 0 7 5 3 1 8 6 4 2 0
0 2 4 6 8 1 3 5 7 0 2 4 6
L ← R
2/0 8/3 14/6
6/2 12/5 18/8
4/1 10/4 16/7
o o u o u u o o u o u u o o u o
u o o u o u u o o u o u u o o u
1 7/2 13/5
3/0 9/3 15/6
5/1 11/4 17/7
L → R

FIG.3

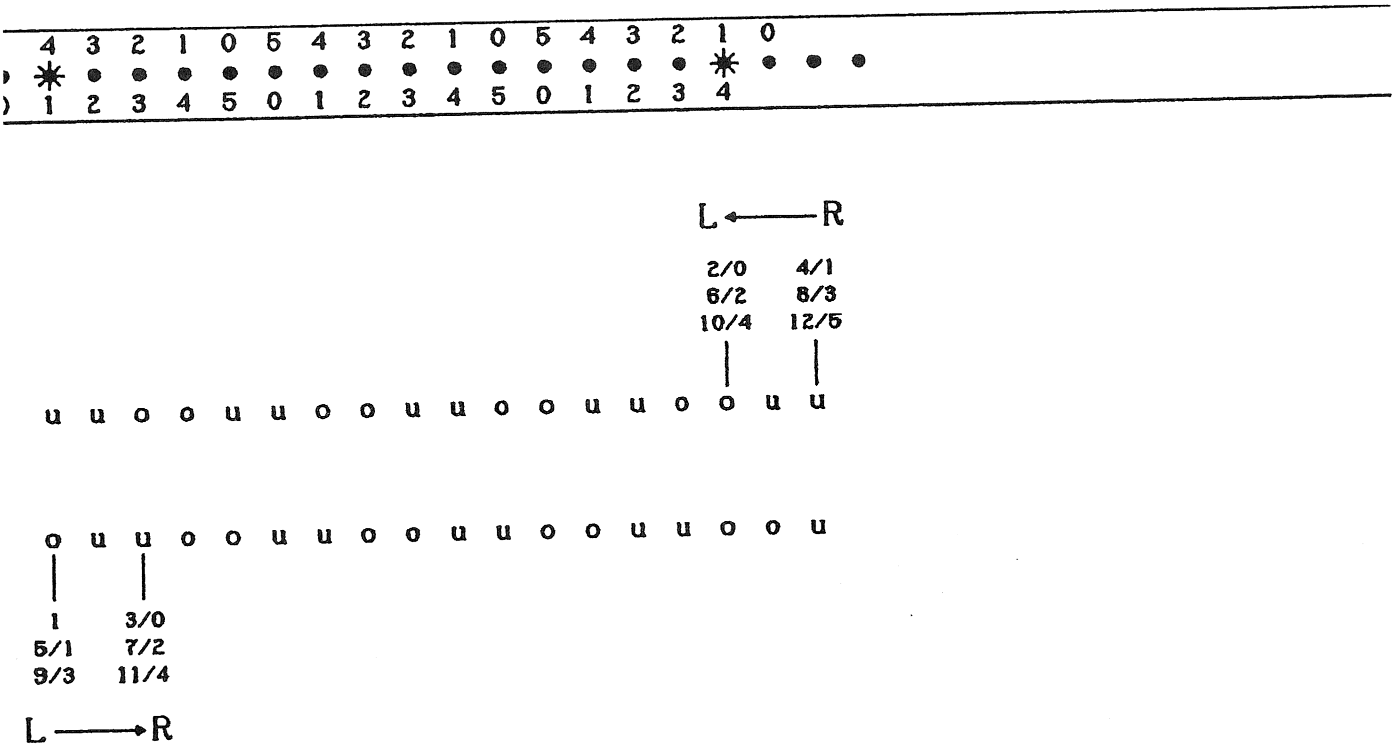

4 3 2 1 0 5 4 3 2 1 0 5 4 3 2 1 0
1 2 3 4 5 0 1 2 3 4 5 0 1 2 3 4
L←R
2/0
6/2
10/4
4/1
8/3
12/5
u u o o u u o o u u o o u u o o u u
o u u o o u u o o u u o o u u o o u
1
5/1
9/3
3/0
7/2
11/4
L→R

FIG. 4

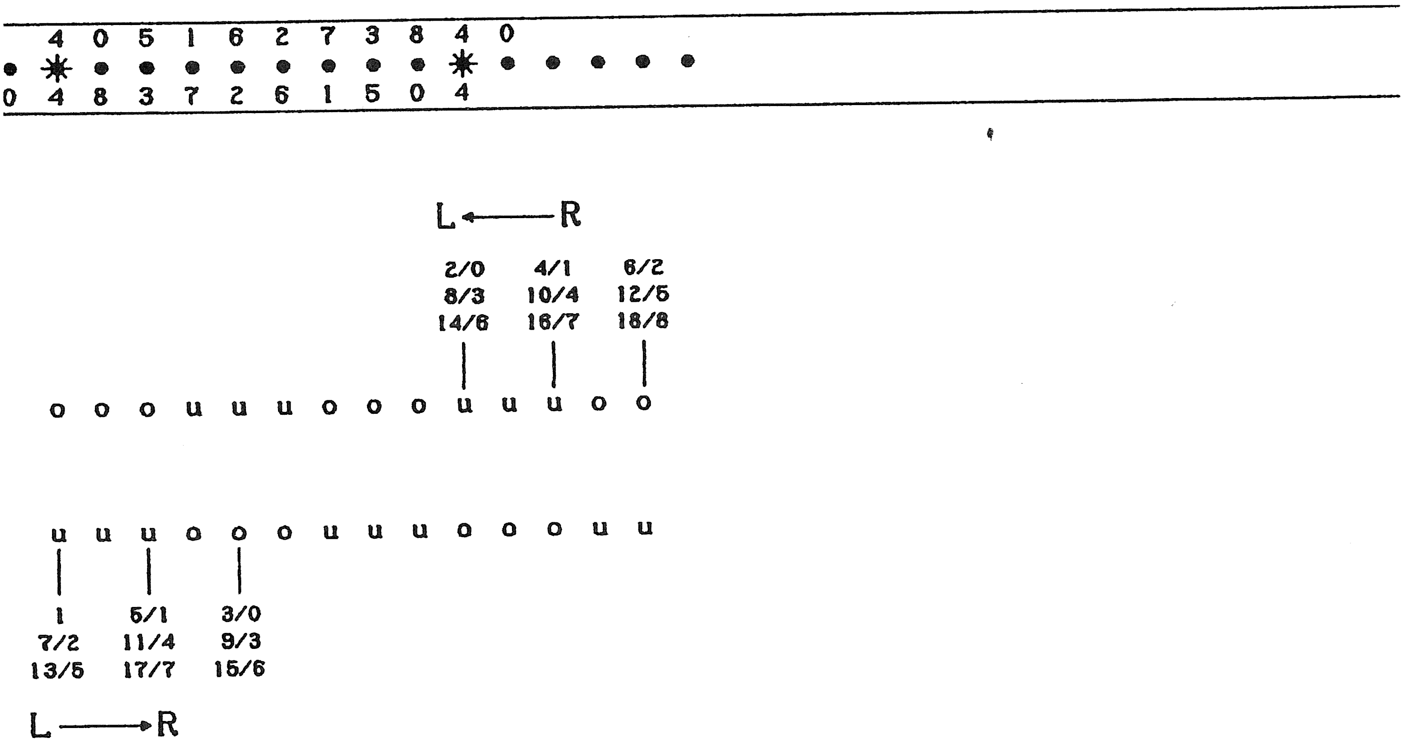

4 0 5 1 6 2 7 3 8 4 0
0 4 8 3 7 2 6 1 5 0 4
L←R
2/0
8/3
14/6
4/1
10/4
16/7
6/2
12/5
18/8
o o o u u u o o o u u u o o
u u u o o o u u u o o o u u
1
7/2
13/5
5/1
11/4
17/7
3/0
9/3
15/6
L→R

FIG. 5

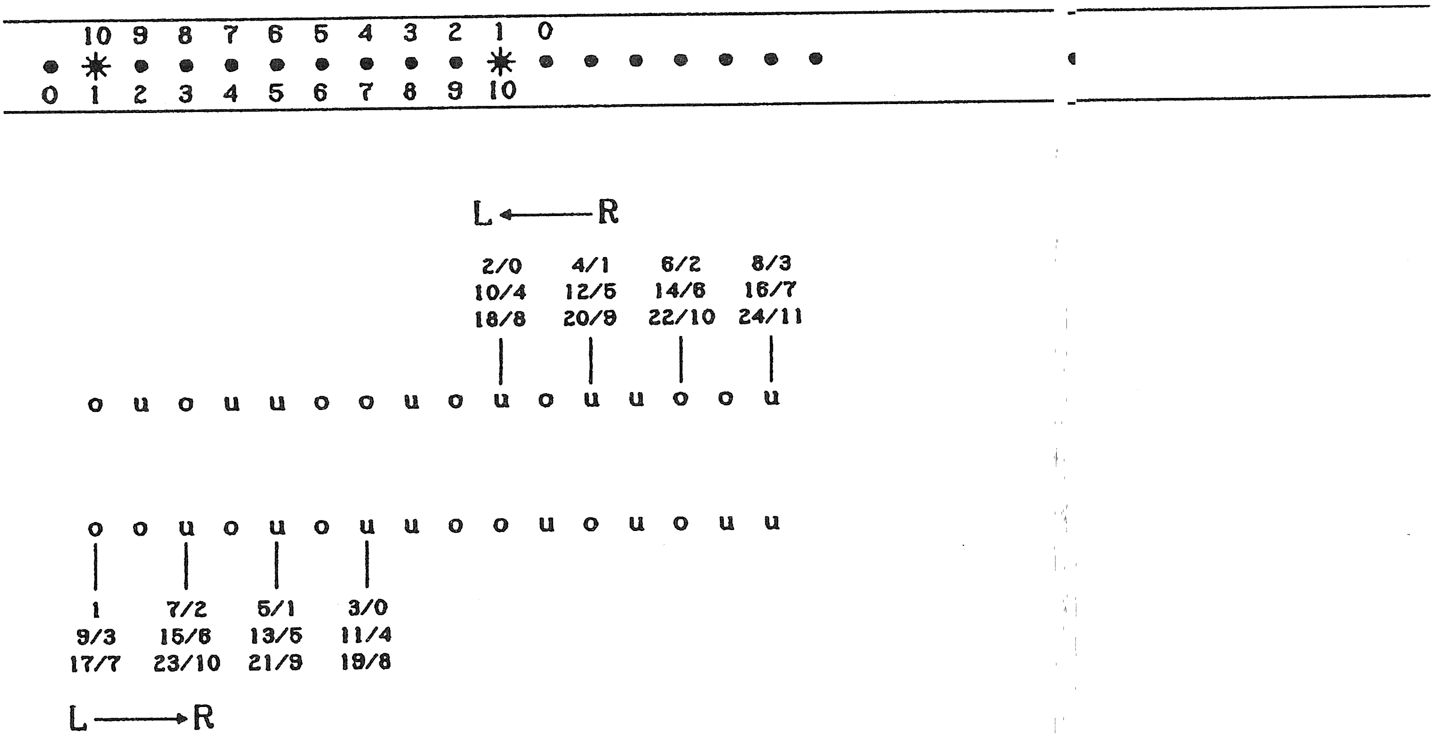

10 9 8 7 6 5 4 3 2 1 0
0 1 2 3 4 5 6 7 8 9 10
L←R
2/0 10/4 18/8
4/1 12/5 20/9
6/2 14/6 22/10
8/3 16/7 24/11
o u o u u o o u o u o u u o o u
o o u o u o u u o o u o u o u u
1 9/3 17/7
7/2 15/6 23/10
5/1 13/5 21/9
3/0 11/4 19/8
L→R

FIG.6

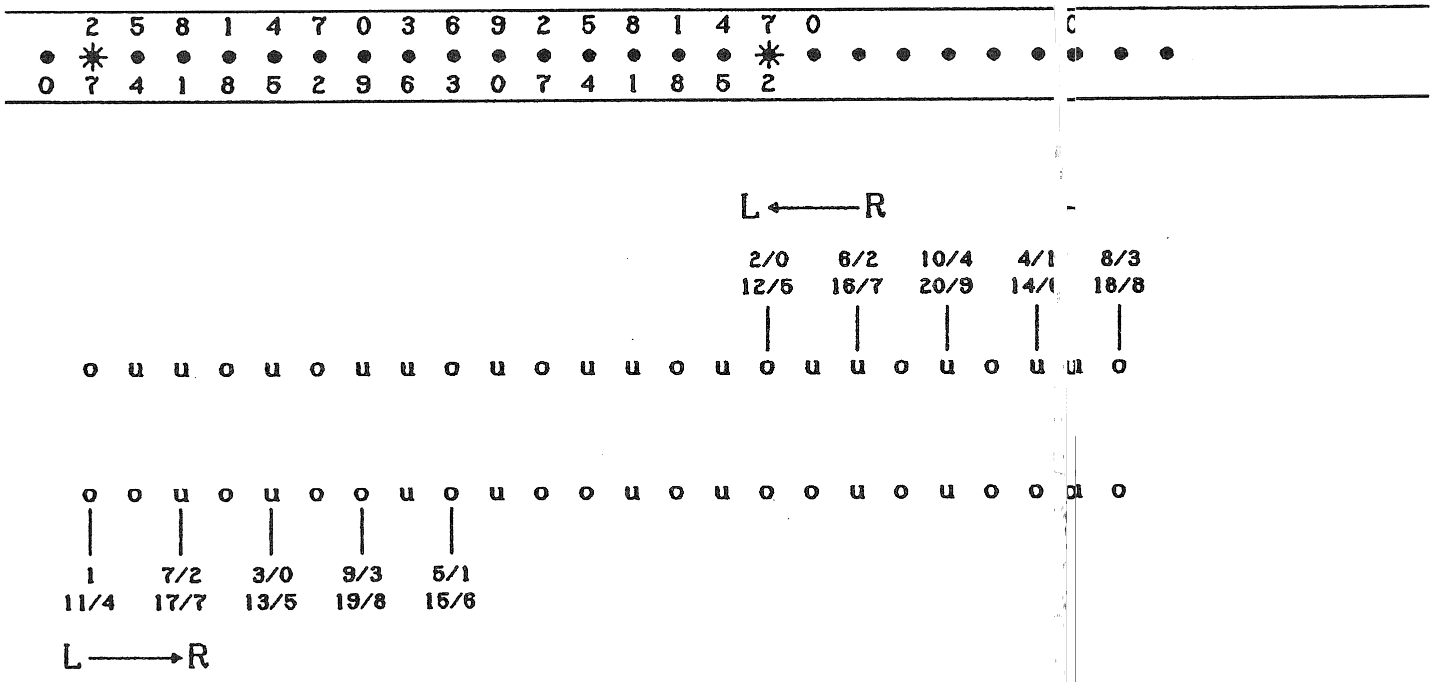

2 5 8 1 4 7 0 3 6 9 2 5 8 1 4 7 0
0 7 4 1 8 5 2 9 6 3 0 7 4 1 8 5 2
L←R
2/0 12/5
6/2 16/7
10/4 20/9
4/1 14/(
8/3 18/8
o u u o u o u u o u o u u o u o u u o u o u u o
o o u o u o o u o u o o u o u o o u o u o o u o
1 11/4
7/2 17/7
3/0 13/5
9/3 19/8
5/1 15/6
L→R

FIG.7

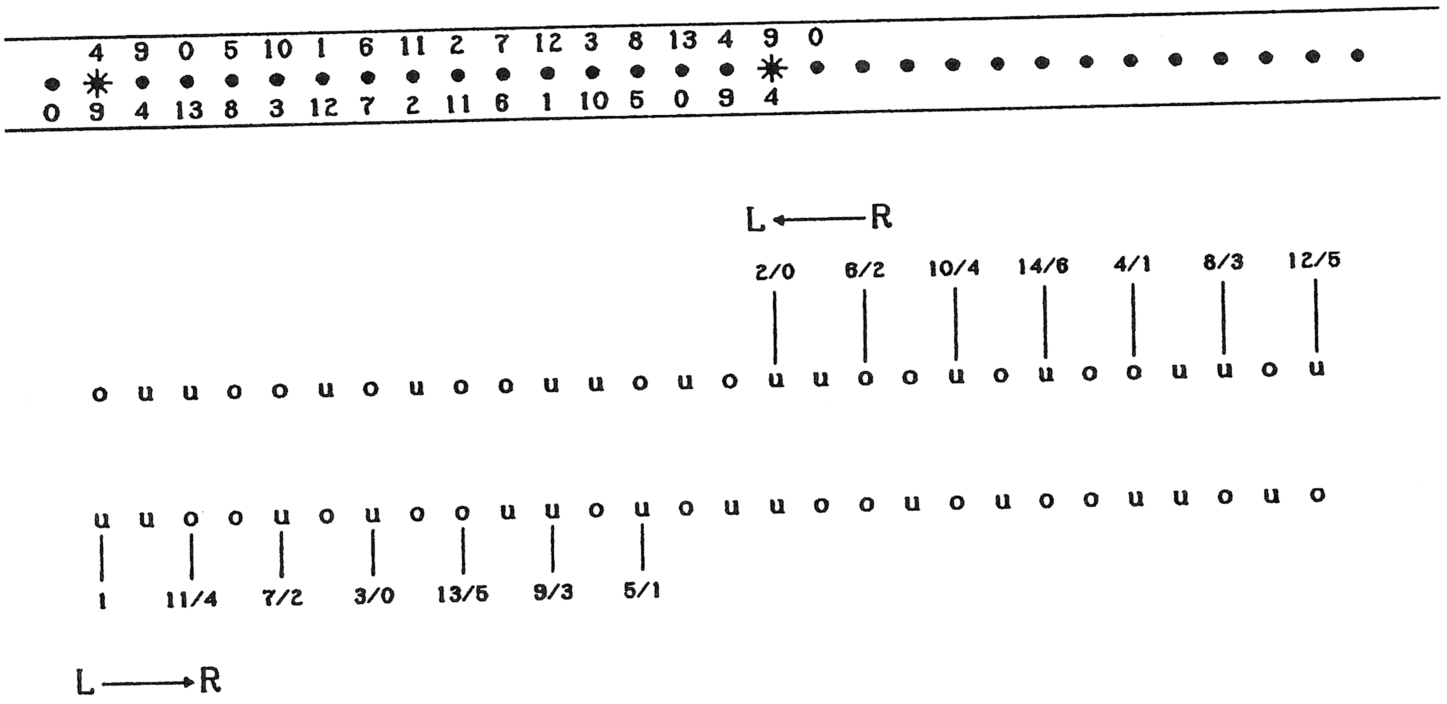


FIG.8 — upwards

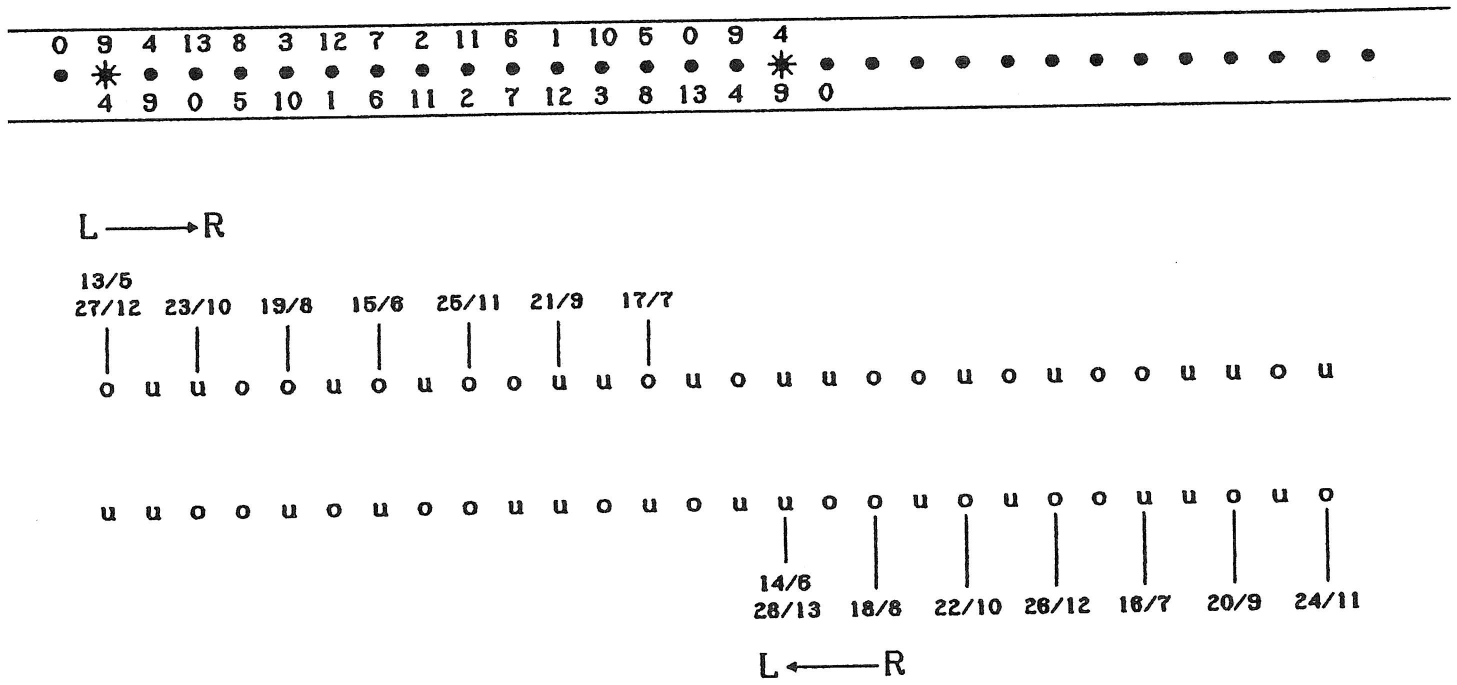


FIG.8 — downwards

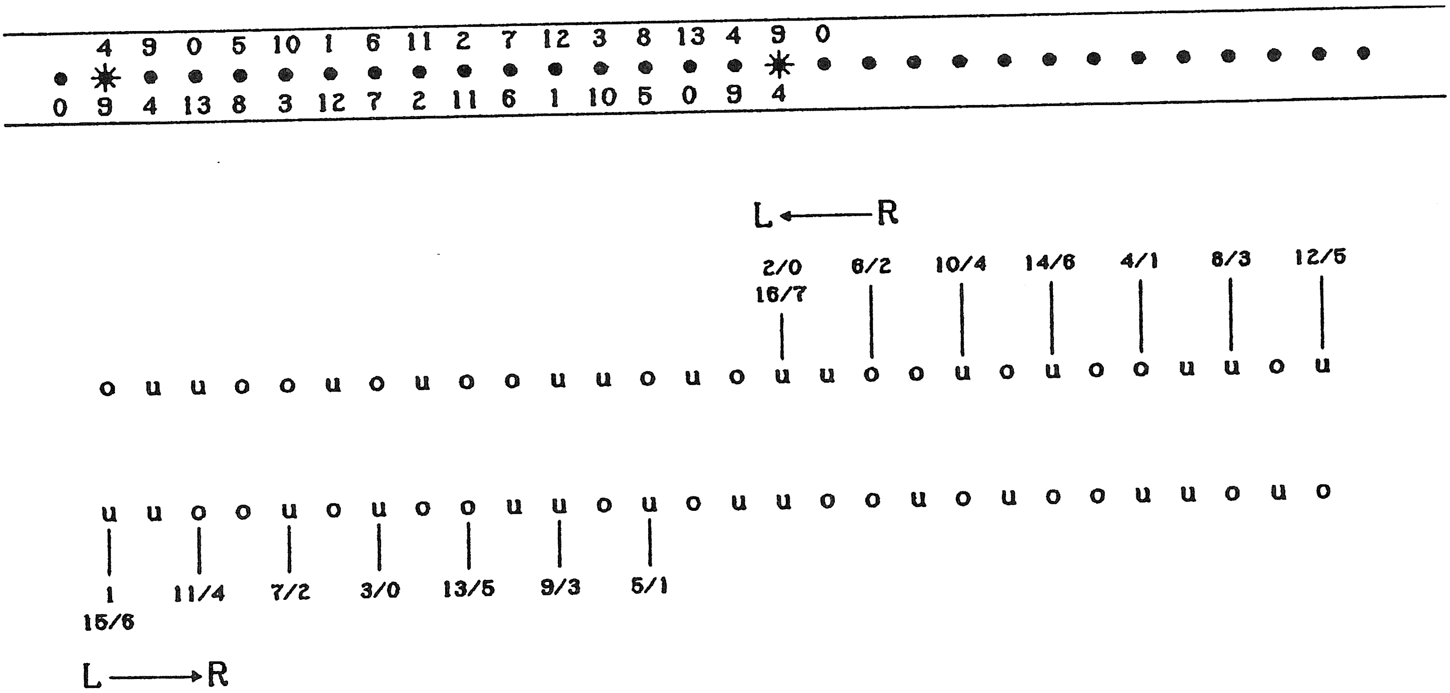


FIG.9 — upwards

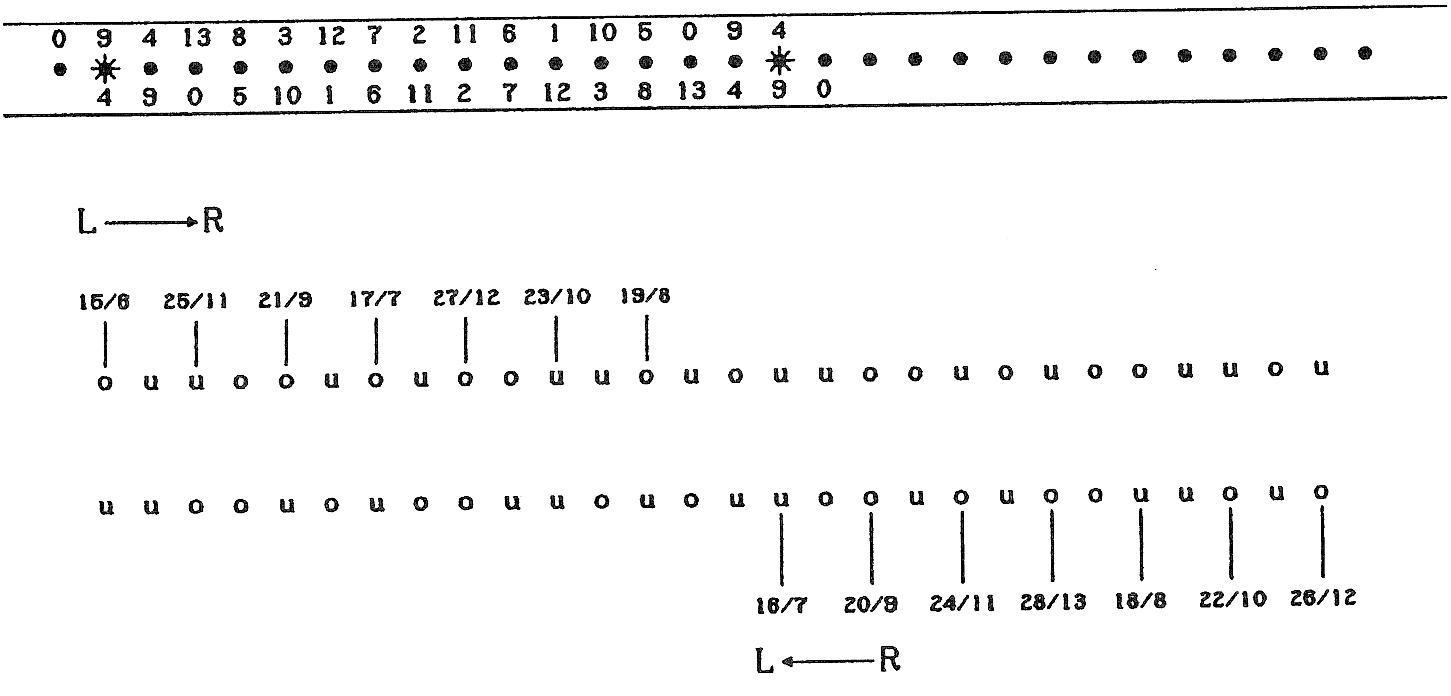


FIG.9 — downwards

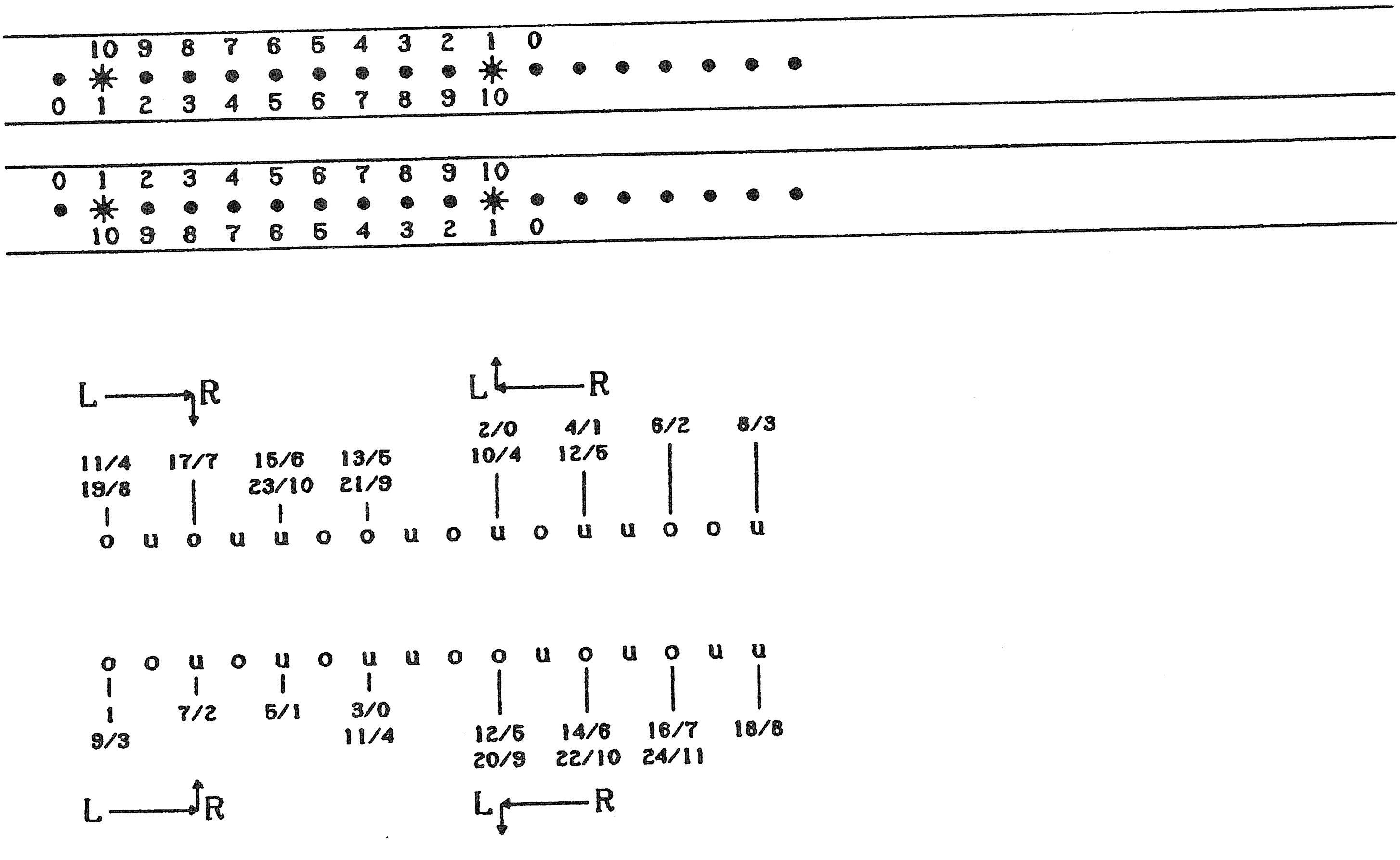
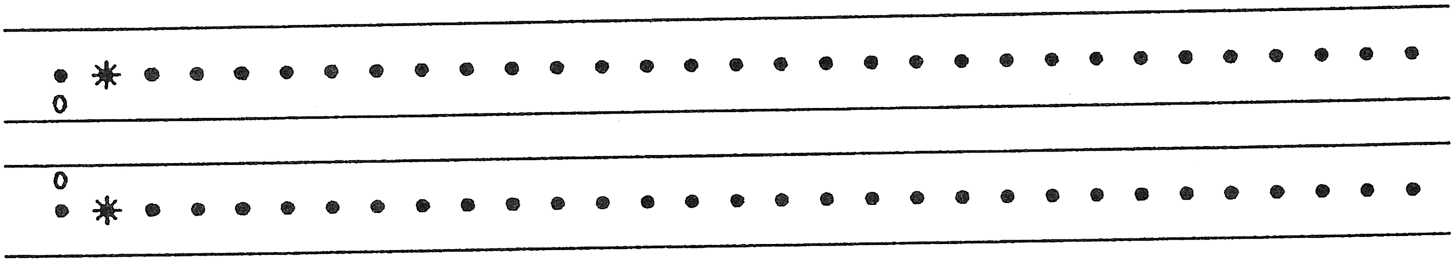

10 9 8 7 6 5 4 3 2 1 0
0 1 2 3 4 5 6 7 8 9 10
0 1 2 3 4 5 6 7 8 9 10
10 9 8 7 6 5 4 3 2 1 0
L R
L R
11/4 19/8
17/7
15/6 23/10
13/5 21/9
2/0 10/4
4/1 12/5
6/2
8/3
o u o u u o o u o u o u u o o u
o o u o u o u u o o u o u o u u
1 9/3
7/2
5/1
3/0 11/4
12/5 20/9
14/6 22/10
16/7 24/11
18/8
L R
L R

FIG.10


0
0